Idle Liquidity, CBDC and Banking *

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Abstract

We build models with an interest-bearing central bank digital currency (CBDC) to investigate whether the interest-bearing CBDC can lead to financial disintermediation. Our model of CBDC and banking captures a key feature of an intermediated CBDC system that idle CBDC can be converted into deposits. Higher interest rates on CBDC could increase bank lending and investment in a CBDC-only economy, because in the presence of idle liquidity, CBDC and bank deposits are complements. The higher return on CBDC encourages entrepreneurs to accumulate more CBDC and then deposit more. The interest rate on reserves and the reserve requirement ratio can be effective policy tools that affect bank lending and investment. Our quantitative analysis shows that a higher CBDC interest rate always promotes investment and its effect on lending depends on the type of banking equilibrium. We consider extensions where cash and interest-bearing CBDC can coexist. The coexistence may require the central bank to adjust the CBDC interest rate or the interest rate on reserves. Our results suggest that the design of CBDC and banking are crucial for understanding the effects of CBDC on banking and the macroeconomy.

Key words: CBDC, banking, idle liquidity, interest rates, investment

JEL Classification: E50, E58, E40

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1 Introduction

The recent development of central bank digital currency (CBDC) has sparked a growing literature investigating its macroeconomic implications.[1] China is a pioneer in experimenting CBDC, and has intensively run pilot projects in multiple cities since late 2019. It even launched an e-CNY mobile application in January 2022, which now can be used for payment in 17 Chinese provinces/cities with pilot projects of e-CNY. Furthermore, the pilot projects have revealed major features of e-CNY: first, it belongs to M0, the same as cash in circulation, except in a digital form; second, it is distributed through an intermediated or two-tier operational system, where the central bank stays at the first tier, with commercial banks handling retail payments at the second tier; third, it is a hybrid account-based and token-based payment system, and will coexist with cash for a long time (PBOC, 2021). These major features also reflect mainstream design features of CBDC among other central banks. Particularly, the 2022 BIS survey documents that the intermediated system is not only the arichitecuture of China's CBDC, but also considered by more than 70% of central banks engaged in some form of CBDC work (Kosse and Mattei, 2022).[2]

The digital form of CBDC implies that it can be designed to bear interest, and the interest rate of CBDC can be positive, zero or negative. The potential interest-bearing feature raises two concerns about CBDC: first, whether CBDC would cause financial disintermediation by competing with deposits, and second, how CBDC would coexist with cash. Our paper addresses these concerns by building a model of CBDC and banking where idle CBDC can be converted into bank deposits, *capturing a key aspect of the intermediated system of CBDC*. In such a system, the digital wallet/accounts of CBDC are opened through commercial banks at the second tier and are linked with bank accounts to facilitate fast conversion between CBDC and bank deposits (see Appendix A for more details of this system, represented by China's CBDC). Incorporating this feature also distinguishes our model from existing models of CBDC (Andolfatto 2021, Keister and Sanches, 2023; Chiu et al., 2023) and sheds light on the concerns from a new perspective.

We start from a benchmark model with CBDC and no cash, where we explicitly model a

[1] The 2022 survey of Bank for International Settlements (BIS) shows that more than 90% of central banks are actively engaged in some form of CBDC work and the share of central banks currently developing a CBDC or running a pilot doubled from 14% to 25% (Kosse & Mattei, 2022).

[2] Another good reference is the CBDC of Nigeria, the eNaira, which has also adopted an intermediated arichtechture (CBN, 2021). Nigeria is the biggest economy in Africa, and has formally launched the eNaira, since October 2021.

frictional deposit market and a frictional loan market to incorporate banking and investment. Entrepreneurs hold CBDC, and may or may not have investment opportunities. If they do not have investment opportunities (labelled as type-0), they deposit idle CBDC at banks in a frictional deposit market. If they do (labelled as type-1), they use CBDC as a down payment to apply for bank loans in a frictional loan market, to acquire capital and produce the final output. We consider four policy tools. The first one is a traditional monetary policy tool of changing the growth rate of the money supply (equivalent to changing the inflation rate at steady states). The second is a new tool of changing the CBDC interest rate. Banks in our model are subject to a reserve requirement, and both CBDC and reserves can be used to satisfy the reserve requirement. Hence, we can consider two additional policy tools: changing the reserve ratio and changing the interest rate on reserves.

There are *two* main results from the benchmark model. The *first* result is that a higher CBDC interest rate does not necessarily lead to financial disintermediation and hurt investment. Instead, it promotes investment and in some equilibria raises bank lending. The mechanism behind this result is novel, in comparison with existing models of CBDC where CBDC and bank deposits are substitutes. For example, in Keister and Sanches (2023), a higher CBDC interest rate crowds out deposits, which reduces bank lending and investment. Andolfatto (2021) assumes that individuals hold deposits at private accounts of a monopolist bank, but hold CBDC at public accounts. The bank either sets the deposit rate to match the CBDC interest rate to retain deposits or directly borrow from the central bank. In both cases, the lending rate is independent of the deposit rate, a higher CBDC interest rate does not reduce bank deposits and lending. Similarly, Garratt et al. (2022) develop a model with large and small banks and find that a higher CBDC interest rate leads to a higher deposit rate, but its effect on lending is ambiguous. In Chiu et al. (2023), owing to the imperfect competition in the deposit market, a higher CBDC interest rate may help limit banks' market power and force banks to offer a higher deposit rate. Therefore, deposits and loans increase in response to the higher CBDC interest rate. Brunnermeier and Niepelt (2019) show that the introduction of CBDC could adversely affect deposits, but the simultaneous expansion of the central bank's liabilities leads to changes only in the composition of banks' liabilities without affecting the lending capacity of banks under certain conditions.

In our model, CBDC and banks are complements in the spirit of Berentsen et al. (2007). The investment shock acts as a liquidity shock that makes type-0 entrepreneurs have idle liquidity and type-1 entrepreneurs need liquidity. Banks help channel the idle liquidity from type-0 to type-1 entrepreneurs. The complementarity between CBDC and bank deposits makes a higher CBDC interest rate more favorable to deposits and investment. Therefore, a *key* message from the benchmark model is that the relationship between CBDC and banking matters when assessing the macroeconomic effects of the CBDC interest rate.

The *second* result is that the interest rate on reserves and the reserve requirement ratio can be independent policy tools. When the interest rate on reserves is greater than the CBDC interest rate, banks hold reserves and a higher interest rate on reserves raises the deposit rate, which encourages entrepreneurs to hold more CBDC and promotes investment. In practice, the interest rate on reserves serves as the lower bound in a channel system or floor system. We find that it has a potential role in affecting bank lending and investment through affecting the return on deposits. The reserve ratio affects the general equilibrium allocation only when the reserve constraint binds. Our quantitative analysis shows that a higher reserve ratio causes banks to issue fewer loans and discourages investment.

To understand how cash and CBDC interact, we extend the benchmark model by adding cash to the portfolio of entrepreneurs.[3] This captures, at least, the initial stage of issuing CBDC, when cash and CBDC coexist. We consider two special scenarios. In the first scenario, banks can accept only cash as deposits and use either cash or reserves to satisfy the reserve requirement. Banks can help entrepreneurs store CBDC, but cannot use it in other operations. This assumption resembles the role of CBDC in Andolfatto (2021), where CBDC is stored in a public bank account and cannot be used by private banks. In the second scenario, banks can accept only CBDC as deposits and use cash, CBDC or reserves to satisfy the reserve requirement. This captures the features of fast-growing Internet banks or the online banking business operated by traditional banks, where banks mainly deal with digital/electronic money, and do not accept cash from customers.

In both scenarios, cash and CBDC can coexist in all general equilibria, even when CBDC has a non-zero interest rate. In some equilibria, the coexistence requires the central bank to give up either the CBDC interest rate or the interest rate on reserves as an independent policy tool.

[3] Our paper focuses on entrepreneurs' cash holding because corporate cash holding has been an important issue for firms in the U.S. and other advanced economies since the 1980s (see Bates et al. 2009, Azar et al. 2015, Graham and Leary 2018, among others). Graham and Leary (2018) document that the level of average cash holdings is around 25% of assets for U.S. firms.

When the reserve constraint binds, cash and CBDC can coexist without sacrificing any policy tools. Our results on coexistence demonstrate how coexistence can be achieved by considering economic tradeoffs between cash and CBDC without the need to assume limited participation or segmented markets. The extensions also highlight the importance of the relationship between CBDC and banking in understanding the effects of CBDC. In the first scenario where cash and banking are complements, a higher CBDC interest rate does cause financial disintermediation as entrepreneurs switch from cash to CBDC and fewer cash holdings reduce bank lending. However, in the second scenario, CBDC and banking become complements and our result confirms the finding in the benchmark model that CBDC may not cause financial disintermediation.

Our paper produces *three main contributions* to the fast growing literature on CBDC. The *first* contribution is that we model CBDC and bank deposits as complements, capturing features in the mainstream CBDC operational system across various countries, and find that a higher CBDC interest rate does not always lead to financial disintermediation. Theoretically, our paper and the existing papers such as Keister and Sanches (2023) represent two main ways to model money and banking.[4] More importantly, our modelling choice sheds light on the mainstream intermedidated operational system of CBDC.[5] A critical feature of this system is to allow idle CBDC to be converted to bank deposits, which is well captured in our model. This complementarity between CBDC and banking leads to the novel result that a higher CBDC interest rate does not crowd out deposits. Hence, our findings lend support to the mainstream CBDC operational system.

The *second* contribution is that we show the coexistence of cash and CBDC can be achieved by using appropriate policy tools. For any country that considers issuing CBDC, CBDC will be likely to coexist with cash, due to the consideration such as financial inclusion, disaster backup and others. Most papers on CBDC assume limited participation or segmented markets to have both assets coexist. We provide new theoretical results on their coexistence through focusing on economic tradeoffs between the two assets.

The *third* contribution is that building on Rocheteau et al. (2018b), we develop a new framework

[4]These two main ways to model money and banking, are best summarized in the survey by Lagos et al. (2017), "in some of these, money and banking are complements, since a bank is where one goes to get cash; in others, they are substitutes, since currency and bank liabilities are alternative payment instruments, allowing one to discuss not only currency but also checks or debit cards."

[5]In addition to Kosse and Mattei (2022), Soderberg et al. (2022, IMF) review six advanced CBDC projects, which either have issued CBDC or have undertaken extensive pilots or tests, and point out the main operational system has converged to an intermediated CBDC system.

with frictional deposit/loan markets and entrepreneurs' financing choices, to assess the impacts of introducing interest-bearing CBDC on banking and firm investment. Banks in Rocheteau et al. (2018b) only issue loans. We add a traditional function of banks, accepting deposits, to capture a realistic feature of banking: banks can channel idle liquidity to investment.

Our paper is mainly related to two lines of literature. The first line is the literature on CBDC, which include the papers discussed earlier and many policy reports including but not limited to Bordo and Levin (2017) and Berentsen and Schar (2018). While the papers discussed earlier address the impact of CBDC on banking, there are also interesting findings about CBDC and welfare. Keister and Sanches (2023) study the effects of introducing CBDC on interest rates, economic activity and welfare. Their results show that introducing CBDC can promote efficiency in exchange and raise welfare. Williamson (2022) finds that central bank issuing CBDC can reduce inefficiency in the private banking system and improve welfare. These findings offer new insights on the essentiality of CBDC, which is not the focus of our paper.

The second line is the banking literature. There are many papers on banking since the canonical paper of Diamond and Dyvbig (1983).[6] Here we list a few that are highly related to our paper. Banks in our models accept idle liquidity as bank deposits and then make loans to those who need liquidity. This is the key mechanism to make CBDC and bank deposits become complements. The role of banks is similar to Berentsen et al. (2007). However, they focus on households' portfolios, their model does not have capital and investment, and the banking sector is competitive. In contrast, we focus on firms' financing decisions, where capital and investment are key choices, and the banking sector is frictional. Our paper shares a similar frictional loan market as Rocheteau et al. (2018b), which address the effects of monetary policy from a corporate finance perspective. However, we incorporate a frictional deposit market into their framework, and focus on CBDC and banking.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 introduces the benchmark model. The policy analysis for the benchmark model is in Section 4. We calibrate the benchmark model and provide numerical analysis in Section 5. Section 6 extends the benchmark model by adding cash, and considers two scenarios: cash only deposits and CBDC only

[6]Some recent papers that study banking include Williamson (2012), Gu et al. (2013), Brunnermeier and Sannikov (2016), Dong et al. (2021), etc.

deposits. Finally, Section 7 concludes the paper.

2 Environment

Time is discrete and continues forever. Each period consists of three stages: Stage 1 is a decentralized deposit market; Stage 2 has a decentralized loan market, and a competitive capital market operating in parallel; and Stage 3 is a centralized market (CM). There are three types of agents: entrepreneurs (e), suppliers (s) and banks (b). There is a measure one of entrepreneurs, who are subject to an investment shock. With a probability n, $n > 1/2$, an entrepreneur has an investment opportunity and needs to acquire capital for production. With the remaining probability $1 - n$, the entrepreneur does not have an investment opportunity. We label them as type-1 and type-0 entrepreneurs, respectively. The investment shock is realized at the beginning of each period. Suppliers can provide capital in the capital market. As in Rocheteau et al. (2018b), the measure of suppliers is irrelevant due to constant returns. There is a measure one of banks that need to first take deposits in the deposit market, satisfying a reserve requirement, and then issue loans in the loan market. Banks are owned by all entrepreneurs equally. All agents discount across periods at rate β, i.e., between the Stage 3 and the next Stage 1.

In the benchmark model, we assume that the central bank issues CBDC m_c but not cash. This describes the scenario when CBDC completely phases out cash. We consider the coexistence of cash and CBDC in Section 6. CBDC is a fiat digital money with the price ρ, measured by the CM numeraire goods x. It is interest-bearing with a nominal interest rate i_c paid every period. The timeline of a representative period is shown in Figure 1, and the details of each stage are as follows.

At Stage 1, after the investment shock is realized, all type-0 entrepreneurs deposit their idle balances at banks. The deposit market allows entrepreneurs to convert their idle CBDC into deposits, capturing a key feature of an intermediated CBDC design. We assume a simple matching technology in the deposit market: short-side being served. Given the measure of type-0 entrepreneurs is $1 - n$, the matching probability for a type-0 entrepreneur is 1 and that for a bank is $1 - n$. Banks without deposits will not proceed to the loan market, due to a reserve requirement which requires banks to hold a fraction υ ($0 < \upsilon < 1$) of total deposits in the form of reserves assets.[7] Both

[7]Notice that, to model the reserve requirement, Rocheteau et al. (2018b) introduce an interbank market where banks can borrow at a policy rate, while we introduce a frictional deposit market.

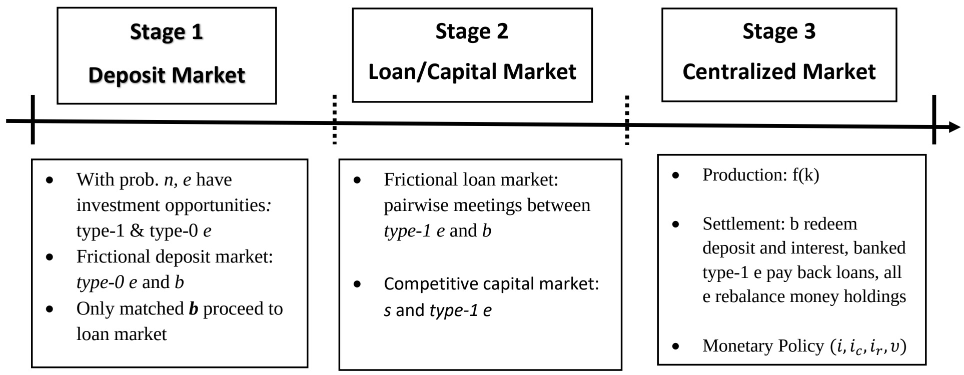

Figure 1: Timeline of a Representative Period

CBDC and reserves are liquid assets that can satisfy the reserve requirement. In contrast, CBDC in Andolfatto (2021) can only be stored at banks. Banks and entrepreneurs bargain over the terms of the deposit contract.

Banks obtaining deposits and all type-1 entrepreneurs participate in Stage 2. We again use the simple matching technology: short-side being served. Given that the measure of type-1 entrepreneurs is n and that of banks is $1-n$, the matching probability for a type-1 entrepreneur is $(1-n)/n$ and that for a bank is 1 since $n > 1/2$. In the loan market, we adopt the banking arrangement with circulating banking liabilities as in Rocheteau et al. (2018b). When a bank makes a loan to an entrepreneur, the bank credits the entrepreneur's deposit account by the amount of the loan. The entrepreneur then uses the deposit claim to purchase capital from suppliers. Banks and entrepreneurs bargain over the terms of the loan contract, including a down payment p (in the form of CBDC), a loan service fee ϕ and a loan size ℓ. Banked type-1 entrepreneurs use CBDC as down payment and purchase capital with both internal and external finance at the market price q_k as suppliers produce capital using their own labor. To ensure the repayment of loans, we assume that a fraction χ of the entrepreneur's output is pledgeable. It implies that banks can seize a fraction χ of output in the case of default. As for unbanked type-1 entrepreneurs, i.e., those who do not get loans from banks, they use internal finance to purchase capital from suppliers.

At Stage 3, all agents participate in the competitive CM. Entrepreneurs who deposit at Stage 1 redeem their deposits, and entrepreneurs who borrow in the loan market repay the loans and banking service fees. Banks distribute all profits to entrepreneurs. Entrepreneurs use capital k

to produce, with the technology $f(k)$, where $f'(k) > 0$, $f''(k) < 0$ and $f(k)$ satisfies the Inada conditions. All agents can consume the numeraire good x in the CM. A negative x implies that agents work and produce x.

The government is active only at Stage 3. It is a consolidated monetary and fiscal authority, and fiscal policy passively accommodates any changes from monetary policy. We consider four policy tools. Let M be the amount of CBDC in the end of Stage 3. The first policy tool is to change the growth rate of CBDC μ, measured by $M/M_{-} = 1 + \mu$, where $1 + \mu \equiv \rho/\hat{\rho}$ in the steady state and μ is the inflation rate. We use $\hat{\rho}$ to denote the price of CBDC in the next CM. The Fisher equation implies that $1 + i = (1 + \mu)/\beta$.[8] The second policy tool is to set the interest rate on CBDC, i_c, with $i_c \leq i$ due to the no-arbitrage condition, but it is possible to have $i_c < 0$. When $i_c < 0$, the central bank implements a negative interest rate (NIR) policy. The third policy tool is to set the interest rate on reserves, i_r, again with $i_r \leq i$ due to the no-arbitrage condition. Banks have access to reserves issued by the monetary authority. Paying interest on reserves has become a new monetary policy tool for a number of central banks including the Bank of England and the US Federal Reserve since the Great Recession. The last policy tool is that the central bank can change the reserve requirement ratio υ, $0 < \upsilon < 1$.

Since banks distribute all profits in Stage 3, there is no reserve in the economy in the end of each period. Banks accept deposits as their liabilities and decide whether to hold CBDC or reserves as their assets in Stage 1. Then banks can expand their balance sheets through issuing loans in Stage 2. Let (M_c, M_r) be the amounts of CBDC and reserves in the beginning of Stage 3 in the current period, where $M = (1 + i_c) M_c + (1 + i_r) M_r$ and (i_c, i_r) are the interest rates on CBDC and reserves. We use the subscript "$-$" to denote variables associated with the previous period. The budget constraint of the government is,

$$G + T = \rho(M - M_{-}) - \rho i_c M_c - \rho i_r M_r, \tag{1}$$

where G is government spending and T is lump-sum transfers in real terms. The LHS in (1) refers to the total government expenditure, while the RHS is the seigniorage revenues net of CBDC and reserves interest payments.

[8]Here i can be interpreted as the nominal interest rate of illiquid bonds, which measure the opportunity cost of holding fiat money with zero interest.

3 Model

In the benchmark model, we use U^j, V^j and W^j to denote the value functions for a type-j agent at Stages 1, 2 and 3, where $j = \{e, b, s\}$. For $j = e$, we have U_i^e, V_i^e and W_i^e for $i = \{0, 1\}$, to differentiate type-0 and type-1 entrepreneurs once the investment shock is realized in the beginning of Stage 1.

We start from Stage 3 in the current period, followed by Stages 1 and 2 in the next period. In the beginning of Stage 3, there are two types of entrepreneurs determined by the realized investment shock at Stage 1. For a type-1 entrepreneur,

$$W_1^e(z_c, p_\ell, k) = \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\}$$
$$\text{st. } x + (1+\mu)\hat{z}_c = (1+i_c)z_c - p_\ell + f(k) + T + \Pi,$$

where $z_c = \rho m_c$ is the real balance of CBDC, $\hat{z}_c = \hat{\rho}\hat{m}_c$ is the real balance of CBDC carried to next period, $p_\ell \equiv \ell + \phi$ is the total loan repayment, including ℓ the amount of loans incurred in the previous loan market and ϕ the banking service fee, $f(k)$ is the final output with capital k, and (T, Π) represent transfers from the government and profits distributed by banks. Substituting x from the budget constraint, we have

$$W_1^e(z_c, p_\ell, k) = (1+i_c)z_c - p_\ell + f(k) + T + \Pi + \max_{\hat{z}_c}\{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c)\}.$$

A type-0 entrepreneur redeems deposits and chooses the amount of CBDC to carry

$$\begin{aligned} W_0^e(z_c, d) &= \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z}_c &= (1+i_c)z_c + (1+i_d)\, d + T + \Pi, \end{aligned}$$

where d is the real balance of deposits and i_d is the nominal deposit rate. Similarly, we have

$$W_0^e(z_c, d) = (1+i_c)z_c + (1+i_d)\, d + T + \Pi + \max_{\hat{z}_c} \{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c)\}.$$

It is clear that entrepreneurs will choose the same $\hat{z}_c$ independent of their previous types,

$$1 + \mu = \beta \frac{\partial \mathbb{E} U^e (\hat{z}_c)}{\partial \hat{z}_c}. \tag{2}$$

Banks distribute their profits to entrepreneurs, where $\Pi = \sum \Pi_b$ aggregates all profits from active banks in this period. For each bank,

$$W^b (z_c, z_r, p_\ell, d) = (1 + i_c) z_c + (1 + i_r) z_r + p_\ell - (1 + i_d) d + \beta U^b,$$

where (z_c, z_r) denote the real balances of CBDC and reserves. We define $\Pi_b \equiv (1 + i_c) z_c + (1 + i_r) z_r + \ell + \phi - (1 + i_d) d$. Suppliers are active only in Stage 2 and Stage 3. For a supplier, $W^s = \omega + \beta V^s$,where ω is the wealth upon entering the CM from selling capital, and V^s is the value function in the capital market at Stage 2 of next period.

Moving to Stage 1 in the next period, after the investment shock is realized, type-1 entrepreneurs will directly proceed to the loan/capital market at Stage 2 and type-0 entrepreneurs enter the deposit market to deposit their idle balances. For entrepreneurs,

$$\mathbb{E} U^e (\hat{z}_c) = n U_1^e (\hat{z}_c) + (1 - n) U_0^e (\hat{z}_c). \tag{3}$$

where $U_1^e(\hat{z}_c) = V_1^e (\hat{z}_c)$ and $U_0^e (\hat{z}_c) = W_0^e (\hat{z}_c - d, d)$ for $d \leq \hat{z}_c$. For banks,

$$U^b = (1 - n) V^b (z_c, z_r, d) + n V^b (0, 0),$$

since banks without deposits will exit from the market. That is, $V^b (0, 0) = W^b (0) = 0$, with $\Pi_b = 0$.

In the loan market at Stage 2, type-1 entrepreneurs and banks with deposits meet. For a type-1 entrepreneur,

$$V_1^e (\hat{z}_c) = \frac{1 - n}{n} W_1^e \left(\hat{z}_c - \frac{p_b}{1 + i_c}, p_\ell, k_b \right) + \frac{2n - 1}{n} W_1^e \left(\hat{z}_c - \frac{p_m}{1 + i_c}, 0, k_m \right).$$

With probability $(1 - n)/n$, a type-1 entrepreneur successfully matches with a bank and uses down payment p_b, $p_b \leq (1+i_c)\hat{z}_c$ to get a loan ℓ to acquire capital k_b. Hence, the loan size ℓ satisfies $\ell = k_b - p_b$. Notice that the down payment p_b is measured as its value in Stage 3. As mentioned before, the total repayment is $p_\ell = \ell + \phi$. With the remaining probability, the entrepreneur can only resort to internal finance p_m, $p_m \leq (1+i_c)\hat{z}_c$, to acquire capital k_m. Notice the subscripts (b, m) denote terms related to banked and unbanked type-1 entrepreneurs. For a bank,

$$V^b(z_c, z_r, d) = -q_k k_b + W^b\left(z_c + \frac{p_b}{1+i_c}, z_r, p_\ell, d\right), \tag{4}$$

where the first term at the RHS means banks pay the amount of $q_k k_b$ to suppliers on behalf of banked entrepreneurs. As for suppliers in the capital market, $V^s = \max_k\{-k + W^s(q_k k)\}$,which leads to $q_k = 1$.

3.1 Bargaining

After defining the value functions, we consider how the deposit contract and the loan contract are determined. In the deposit market, let γ be the bargaining power of banks with $0 < \gamma \leq 1$. The Nash bargaining problem in the deposit market is

$$\max_{d, i_d} [\phi + (\max\{i_r, i_c\} - i_d) d]^\gamma [(i_d - i_c) d]^{1-\gamma} \text{ st. } d \leq \hat{z}_c. \tag{5}$$

After accepting idle CBDC as deposits, the bank can decide whether to hold CBDC or convert CBDC into reserves.[9] When $i_c \geq i_r$, banks hold CBDC to satisfy the reserve requirement. When $i_c < i_r$, banks instead hold reserves. In either case, the total surplus from the deposit contract is positive and $i_d \geq i_c$. Therefore, we let $d = \hat{z}_c$ as type-0 entrepreneurs weakly prefer to deposit.

In the loan market, the surplus of a type-1 entrepreneur is

$$W_1^e\left(\hat{z}_c - \frac{p_b}{1+i_c}, p_\ell, k_b\right) - W_1^e\left(\hat{z}_c - \frac{p_m}{1+i_c}, 0, k_m\right) = f(k_b) - k_b - \phi - \Delta_m,$$

where $\Delta_m \equiv f(k_m) - p_m$ is the trading surplus for an unbanked type-1 entrepreneur and $p_m = k_m = (1+i_c)\hat{z}_c$. The bank's surplus is ϕ. Let the bank's bargaining power be θ. Taking d and $\hat{z}_c$

[9]CBDC in our model integrates the features of a retail CBDC and a wholesale CBDC. When entrepreneurs use CBDC as downpayment, CBDC functions as a retail CBDC. When banks hold CBDC to satisfy the reserve requirement, CBDC held by banks functions as a wholesale CBDC.

as given, the Kalai bargaining problem is[10]

$$
\begin{aligned}
&\max_{p_b,\phi,k_b} \phi \\
\text{st. } & \phi = \theta \left[f\left(k_b\right) - k_b - \Delta_m \right], && (6) \\
& \ell + \phi \leq \chi f\left(k_b\right), && (7) \\
& \upsilon\left(\ell + d\right) \leq d, && (8) \\
& p_b \leq (1+i_c)\hat{z}_c. && (9)
\end{aligned}
$$

Here, we have $q_k k_b = k_b = p_b+\ell$, since q$_k$ $= 1 withthecompetitivecapitalmarket.Thefirstinequalityconstraint(7)is$ $1 entrepreneur : thetotalamountowedtothebankmustbenomorethanafraction \chi$ of final output $f(k)$ as the collateral. The second inequality constraint (8) is the reserve constraint for the bank: the amount of CBDC/reserves d must be no less than a fraction υ of total deposits $\ell + d$. When issuing loans, the bank simultaneously create deposit claims which can be circulated. This way of modelling the loan arrangement is similar to Rocheteau et al. (2018b), but different from Berentsen et al. (2007), where banks reallocate liquidity among agents. In our model, banks are essential not only because they reallocate liquidity, but also because banks can create liquidity. The third constraint (9) indicates the down payment p_b cannot exceed the value of CBDC. We consider $p_b = (1 + i_c)\hat{z}_c$ because there is no benefit for the type-1 entrepreneur to keep some extra CBDC given that $i_c \leq i$. In what follows, we define $\delta \equiv 1/\upsilon - 1$ as the loan to reserve ratio and use δ to discuss the banking policy since there is a one-to-one relationship between υ and δ.

To solve the deposit contract and the loan contract, we first solve the loan contract and then move back to the deposit contract because the loan contract could potentially depend on the deposit contract. Using $\ell = k_b - p_b$, we set up the Lagrangian,

$$
\begin{aligned}
\mathcal{L}(k_b, \lambda_1, \lambda_2) = & \max_{k_b,\lambda_1,\lambda_2} \theta \left[f\left(k_b\right) - k_b - \Delta_m \right] \\
& - \lambda_1 \left\{ k_b - (1+i_c)\, \hat{z}_c + \theta \left[f(k_b) - k_b - \Delta_m \right] - \chi f\left(k_b\right) \right\} \\
& - \lambda_2 \left[k_b - (1+i_c)\, \hat{z}_c - \delta d \right].
\end{aligned}
$$

[10] We adopt Kalai bargaining, instead of Nash, because the former is analytically more tractable. In previous versions, we try Nash bargaining and it does not change our main results.

The FOCs with respect to $(k_b, \lambda_1, \lambda_2)$ are

$$k_b : \theta[f'(k_b) - 1] = \lambda_1[(\theta - \chi)f'(k_b) + 1 - \theta] + \lambda_2$$

$$\lambda_1 : \lambda_1\{k_b - (1 + i_c)\hat{z}_c + \theta[f(k_b) - k_b - \Delta_m] - \chi f(k_b)\} = 0$$

$$\lambda_2 : \lambda_2[k_b - (1 + i_c)\hat{z}_c - \delta d] = 0,$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Hence, there are three cases to consider.

Case 1: $\lambda_1 = 0$ and $\lambda_2 = 0$. Neither the collateral constraint nor the reserve constraint binds. We have $k_b = k^*$ with $f'(k^*) = 1$ and $\phi = \theta[f(k^*) - k^* - \Delta_m]$.

Case 2: $\lambda_1 > 0$ and $\lambda_2 = 0$. The collateral constraint binds but the reserve constraint does not. We have

$$(\theta - \chi)f(k_b) + (1 - \theta)k_b = k_m + \theta\Delta_m \tag{10}$$

$$\lambda_1 = \frac{\theta[f'(k_b) - 1]}{(\theta - \chi)f'(k_b) + 1 - \theta} > 0 \tag{11}$$

to solve for (k_b, λ_1) and ϕ solves (6).

Case 3: $\lambda_1 = 0$ and $\lambda_2 > 0$. The reserve constraint binds but the collateral constraint does not. We solve for (k_b, λ_2, ϕ) from

$$k_b = \delta d + k_m \tag{12}$$

$$\lambda_2 = \theta[f'(k_b) - 1] > 0, \tag{13}$$

and (6). Notice that a fourth case with $\lambda_1 > 0$ and $\lambda_2 > 0$ is not generically possible because both the collateral and reserve constraints can be used to solve for k_b for given d and $\hat{z}_c$.

For the deposit contract, we consider two possibilities: $i_c \geq i_r$ and $i_c < i_r$. When $i_c \geq i_r$, the bank's balance sheet is described as the top of Figure 2. When $i_c < i_r$, the bank's balance sheet is described as the bottom of Figure 2. The bank's surplus from the deposit contract is $\phi + (\max\{i_r, i_c\} - i_d)\,d$. Since $d = \hat{z}_c$, the FOC with respect to i_d is

$$i_d\hat{z}_c = (1 - \gamma)\phi + [\gamma i_c + (1 - \gamma)\max\{i_r, i_c\}]\,\hat{z}_c. \tag{14}$$

Assets	Liabilities
CBDC	Type-0 entrepreneurs' deposits
Loans to type-1 entrepreneurs	Type-1 entrepreneurs' deposits

Assets	Liabilities
Reserves	Type-0 entrepreneurs' deposits
Loans to type-1 entrepreneurs	Type-1 entrepreneurs' deposits

Figure 2: Bank's Balance Sheet

The deposit rate depends on the size of the deposit $\hat{z}_c$, the banking fee ϕ that the bank earns from the loan contract and the CBDC interest rate i_c. Only when $i_c < i_r$, banks hold reserves and i_r also affects the deposit rate.

3.2 General Equilibrium

The solutions from the deposit contract and the loan contract allow us to solve for $\hat{z}_c$ at Stage 3 using (2) and characterize the general equilibrium.

Definition 1 *Given policy parameters (G, i, i_c, i_r, δ), a stationary monetary equilibrium is a list of $(\hat{z}_c, k_m, k_b, \phi, i_d, d)$ that satisfies [1] bargaining solutions in the loan and deposit markets; [2] entrepreneurs' optimization; [3] the government budget constraint; and [4] all market clearing conditions.*

To determine $\hat{z}_c$, we first derive $\mathbb{E}U^e(\hat{z}_c)$ following (3)

$$\mathbb{E}U^e(\hat{z}_c) = (1-n)(1-\theta)[f(k_b) - k_b - f(k_m) + k_m] + nf(k_m) + (1-n)(1+i_d)\hat{z}_c + nW_1^e(0,0) + (1-n)W_0^e(0,0).$$

The exact expression of $\partial \mathbb{E}U^e(\hat{z}_c)/\partial \hat{z}_c$ depends on whether $i_c \geq i_r$ and whether the constraints in the loan contract bind. From (14), $\partial(i_d \hat{z}_c)/\partial \hat{z}_c = (1-\gamma)\partial\phi^d/\partial\hat{z}_c + [\gamma i_c + (1-\gamma)\max\{i_r, i_c\}]$. We introduce a new notation ϕ^d to represent the banking fee that a depositor's bank earns in the loan market. The depositor's $\hat{z}_c$ and the borrower's $\hat{z}_c$ have different effects on the banking fee. We have three cases to consider for the general equilibrium analysis.

In Case 1, $k_b = k^*$ and $\partial k_b / \partial \hat{z}_c = 0$. The reserve constraint does not bind so that the depositor's $\hat{z}_c$ does not affect ϕ^d. Using (2), we solve for k_m from

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + s_r, \tag{15}$$

where $A \equiv n - (1-n)(1-\theta) > 0$ and $s_r \equiv (1-n)(1-\gamma)\max\{i_r - i_c, 0\}/(1+i_c)$. The term $\max\{i_r - i_c, 0\}/(1+i_c)$ is a spread between the interest rate earned on reserve asset and the CBDC interest rate. In (15), the LHS and RHS represent the cost and benefit of holding an additional unit of CBDC, respectively. The first term of the RHS is the liquidity premium of CBDC and the second term s_r represents the benefits of CBDC derived from the deposit contract. Only when $i_c < i_r$, depositing an additional unit of CBDC could earn more interest. Knowing (k_b, k_m), (6) gives the equilibrium value of ϕ. We label the Case 1 equilibrium as an unconstrained equilibrium. For any given (i, i_c, i_r), this type of equilibrium exists when χ and δ are big enough so that the constraints are slack. That is,

$$\begin{aligned} \chi &> \frac{\theta\left[f\left(k^*\right) - f\left(k_m\right)\right] + (1-\theta)\left(k^* - k_m\right)}{f\left(k^*\right)} \equiv \chi_1 \\ \delta &> \frac{\left(1+i_c\right)\left(k^* - k_m\right)}{k_m} \equiv \delta_1 \end{aligned}$$

where k_m solves (15).

In Case 2, k_b is determined in (10) and ϕ^d does not depend on $\hat{z}_c$. With the binding collateral constraint, we label this type of equilibrium as a collateral constrained equilibrium. From (2), we have

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + \frac{(n-A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'\left(k_b\right) + 1 - \theta}[f'(k_b) - 1] + s_r. \tag{16}$$

When the collateral constraint binds, having an additional unit of CBDC helps relax the collateral constraint and increase the loan size, which further lead to a higher k_b. The second term in the RHS of (16) shows this additional benefit of CBDC. We then solve for (k_b, ϕ) from (10) and (6).

For this type of equilibrium to exist, it requires $\chi < \chi_1$ and

$$\delta \geq \frac{(1+i_c)(k_b - k_m)}{k_m} \equiv \delta_2(\chi)$$

where (k_m, k_b) solves (10) and (16), and depend on χ. In particular, $\lim_{\chi \to 0} \delta_2(\chi) = 0$ and $\delta_2(\chi_1) = \delta_1$.

In Case 3, (12) includes the bank's CBDC d and the type-1 entrepreneur's CBDC $\hat{z}_c$. The binding reserve constraint indicates that the amount of deposits d affects k_b and thus the banking fee depends on the amount of deposits. From the perspective of a type-0 entrepreneur, we have $\partial\phi^d/\partial\hat{z}_c = \theta\delta[f'(k_b) - 1]$. In (12), the type-1 entrepreneur's choice of $\hat{z}_c$ affects k_b directly and $\partial k_b/\partial\hat{z}_c = 1 + i_c$. Again, (2) determine k_m from

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + \left[n - A + \frac{(1-n)(1-\gamma)\theta\delta}{1+i_c}\right][f'(k_b) - 1] + s_r. \tag{17}$$

The binding reserve constraint implies that an additional unit of CBDC has three benefits for entrepreneurs. The first benefit is that it helps relax the reserve constraint and raise the value of k_b, which is reflected by the second term on the RHS of (17). The second benefit is that the additional CBDC helps raise the deposit interest through a higher banking fee. This is reflected by the third term on the RHS of (17). When $i_c < i_r$, $s_r > 0$ reflects the benefit from the deposit contract. Finally, (12) leads to

$$k_b = \left(\frac{\delta}{1+i_c} + 1\right) k_m \tag{18}$$

and ϕ is given by (6). We label this type of equilibrium as a reserve constrained equilibrium. The existence of this type of equilibrium requires $\delta < \delta_1$ and

$$\chi \geq \frac{\theta[f(k_b) - f(k_m)] + (1-\theta)(k_b - k_m)}{f(k_b)} \equiv \chi_3(\delta)$$

where (k_m, k_b) solve (17) and (18), and depend on δ. We can also verify that $\lim_{\delta \to 0} \chi_3(\delta) = 0$ and $\chi_3(\delta_1) = \chi_1$.

As discussed above, the collateral constraint and the reserve constraint cannot bind simultaneously for any parameter values. When they both bind, it requires a specific relationship between χ and δ, which is given by the boundary condition $\delta_2(\chi)$ or $\chi_3(\delta)$. Moreover, $\delta_2(\chi)$ or $\chi_3(\delta)$ are

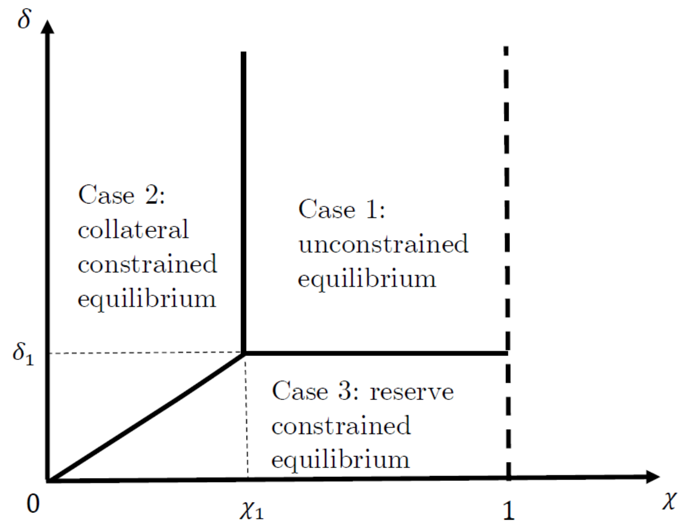

Figure 3: Equilibrium Existence

inverse functions. Figure 3 illustrates the existence of the three types of general equilibrium.

4 Policy Analysis

The equilibrium conditions include four policy parameters (i, i_c, i_r, δ). In this section, we use the benchmark model to analyze how policy affects investment and banking activities. The interest-bearing aspect of CBDC has raised concerns that CBDC may lead to financial disintermediation. In particular, the interest-bearing CBDC can crowd out bank deposits so that banks' lending activities can be adversely affected. This concern views CBDC and bank deposits as substitutes. In our model, banks can take CBDC as deposits and issue loans. Type-0 entrepreneurs have incentives to deposit their idle CBDC, created by the investment shock. The reserve requirement induces banks to take deposits to make loans. CBDC and deposits become complements. We highlight this new perspective of CBDC since our model incorporates features of an intermediated CBDC system and the design of CBDC determines how CBDC and banking activities interact. Through the lens of our model, we examine the effects of the policy parameters (i, i_c, i_r, δ) in each type of equilibrium.

Notice that i_r enters into the equilibrium conditions only when $i_c < i_r$ and δ affects the equilibrium allocation only when the reserve constraint binds. To be more specific, we focus on the effects of policy on four real variables: k_m, k_b, aggregate investment $K = (1-n)\,k_b + (2n-1)k_m$, and aggregate lending $L = (1-n)\,(k_b - k_m)$, and two nominal rates: the deposit rate i_d and the loan rate $i_\ell = (1 + r_\ell)\,(1+\mu) - 1$ where $r_\ell = \phi/\,(k_b - k_m)$.

In the unconstrained equilibrium, a type-1 entrepreneur invests the optimal amount of capital k^*, which is independent of the policy parameters. Since neither the collateral constraint nor the reserve constraint binds, the economy has a loose credit condition. Banked entrepreneurs can borrow to acquire k^*, and banks are not constrained by the reserve holdings when lending to entrepreneurs. We summarize in Proposition 1 the effects of changing i and i_c. Proofs are in Appendix B.

Proposition 1 *In an unconstrained equilibrium: [1] when $i_c \geq i_r$, a higher i_c or a lower i leads to a higher k_m, a higher K, a lower L, and a lower i_ℓ; [2] when $i_c < i_r$, a higher i_c, a higher i_r or a lower i leads to a higher k_m, a higher K, a lower L, and a lower i_ℓ.*

Consider an increase in i_c. It allows a type-0 entrepreneur to afford more k_m. Since the type-1 entrepreneur borrows $k^* - k_m$ from banks, a higher i_c reduces the amount of bank lending and the banking fee. Therefore, r_ℓ and i_ℓ decrease, but i_d depends on both ϕ and i_c. As i_c increases but ϕ decreases, the effect of i_c on i_d is ambiguous. We use calibrated examples in the next section to check how i_d responds to i_c. When $i_c < i_r$, i_r raises the benefit of holding CBDC so the effect of i_r resembles the effect of i_c.

From (15), i and i_c have the opposite effects on the real variables because either a higher i_c or a lower i lowers the cost of holding CBDC. The Friedman rule can be implemented by setting $i - i_c = (1-n)(1-\gamma)\max\{i_r - i_c, 0\}$, which gives the monetary authority flexibility by choosing a combination of (i, i_c, i_r). This is consistent with the finding in Nosal and Rocheteau (2017) on interest-bearing money that the implementation of the Friedman rule does not necessarily require deflation. In terms of welfare, setting $i_c > 0$ does not expand the achievable allocations in the economy. While it is appealing to have theories that rationalize the essentiality of CBDC, our paper focuses on the effect of the CBDC interest rate on banking and investment given the existence of CBDC.

The results for the collateral constrained equilibrium are given in Proposition 2. The binding collateral constraint represents a situation where banks have enough liquidity, but entrepreneurs are collateral-constrained. From (16), a higher i_c allows the type-0 entrepreneur to purchase more k_m and the type-1 entrepreneur to have more down payment. Holding more down payment also enables the type-1 entrepreneur to borrow more from banks, so a higher i_c promotes bank lending and investment. The above result is opposite to the result that CBDC can lead to financial disintermediation, where CBDC competes with deposits and crowds out lending. See Keister and Sanches (2023) for an example. The mechanism that drives our results is also different from the ones in Andolfatto (2021), Garratt et al. (2022) and Chiu et al. (2023). In our model, the conversion of CBDC into deposits make CBDC and banking become complements. The deposit contract ensures $i_d \geq i_c$ so that type-0 entrepreneurs deposit their idle CBDC. Banks use idle liquidity and expand the balance sheet to finance more productive investment.

Proposition 2 *In a collateral constrained equilibrium: [1] when $i_c \geq i_r$, a higher i_c or a lower i leads to a higher k_m, a higher k_b, a higher K, and a higher L; [2] when $i_c < i_r$, a higher i_c, a higher i_r or a lower i leads to a higher k_m, a higher k_b, a higher K, and a higher L.*

Similar to the discussion for the unconstrained equilibrium, i and i_c have opposite effects on the real variables. When $i_c < i_r$, i_r affects the equilibrium allocation and has similar effects on the real variables as i_c does. The effects of the policy parameters on i_d and i_ℓ are ambiguous. This is mainly because when k_m and k_b increase, the banking fee ϕ may or may not increase.

In the reserve constrained equilibrium, only the reserve constraint binds. This when entrepreneurs face loose credit conditions but the banking policy regulates bank lending tightly. As in the other two cases, a higher i_c leads to a higher k_m because it directly benefits entrepreneurs that rely on internal finance. In the reserve constrained equilibrium, k_b is constrained by the amount of CBDC and reserves. A higher i_c indirectly tightens the reserve constraint. The overall effect of i_c on k_b becomes ambiguous. As a result, its effects on investment and lending are ambiguous.

Proposition 3 *In a reserve constrained equilibrium, [1] when $i_c \geq i_r$, a higher i_c leads to a higher k_m; and a higher i leads to a lower k_m, a lower k_b a lower K, a lower L, and a higher i_ℓ; [2] when $i_c < i_r$, a higher i_c leads to a higher k_m; a higher i leads to a lower k_m, a lower k_b a lower K, a lower L, and a higher i_ℓ; and a higher i_r leads to a higher k_m, a higher k_b, a higher K and a higher L.*

It is worth noticing that the effects of i are not opposite to the effects of i_c in this equilibrium. This differs from the findings in the previous two cases. A higher i lowers the return of CBDC and reduces k_m. Since i does not enter into the reserve constraint directly, k_b also decreases. It follows that both K and L decrease. For i_ℓ, both r_ℓ and μ increase, so i_ℓ increases. When $i_c < i_r$, a higher i_r raises i_d, which makes CBDC more appealing. It follows that k_m increases. Since lending is constrained by the amount of deposits, a higher k_m raises lending and k_b. Aggregate investment also increases.

In our model, i_r can play a non-trivial role if $i_c < i_r$. In practice, the interest rate on reserves forms a lower bound for the channel system and the floor system operated by modern central banks. Our model suggests that the interest rate on reserves could have an additional role in affecting bank lending and investment in the economy.

Another interesting observation is that a NIR policy is feasible, i.e., $i_c < 0$.[11] In our environment, $i_c < 0$ is not generally preferable because it reduces entrepreneurs' incentives to hold CBDC. Holding all else equal, this would further lead to less internal finance for unbanked entrepreneurs and less down payment for banked entrepreneurs. However, implementing $i_c < 0$ can be accompanied with appropriate changes in i and i_r to leave the equilibrium allocation unaffected.

5 Quantitative Analysis

Through the analytical results, we demonstrate that a higher CBDC interest rate may not lead to financial disintermediation and reduce investment given that CBDC and banking are complements. In this section, we calibrate our model to the US economy and explore our model's implications. Consider a benchmark model where each period is a year and $i_c = 0$. We adopt the production function $f(k) = k^\omega$, where $0 < \omega < 1$. Following Rocheteau et al. (2018b), we set $\chi = 1$ and $1/\beta = 1.02$, which implies $\beta = 0.9802$. There are seven parameters to calibrate: $(n, \theta, \gamma, \upsilon, \mu, i_r, \omega)$.

The 2019 report on Small Business Credit Survey gives the loan approval rate for small business from 2016 to 2018. The average loan approval rate from large banks and small banks are 75.33% and 69.67%, respectively. The probability for type-1 entrepreneurs to get loans in our model is $(1-n)/n$. Since banks in our model are closer to small banks, we choose n to match the loan approval rate $(1-n)/n = 0.6967$, which gives $n = 0.5894$. Following Chiu et al. (2023), we use

[11] Papers related to negative interest rates include He et al. (2008), Rocheteau et al. (2018a), Dong and Wen (2017), and Groot and Haas (2018). He et al. (2008) and Rocheteau et al. (2018a) use New Monetarist models and can generate negative interest rates for assets. Dong and Wen (2017) and Groot and Haas (2018) study the negative interest rate policy which has happened in some advanced economies (such as Japan, Euro Zone, and some European countries), but neither of them is related to CBDC.

Parameters	Notation	Value	Target
Calibrated externally			
Discount factor	β	0.9802	Rocheteau et al. (2018b): 3-month T-bill rate (real)
CBDC interest rate	i_c	0	set as benchmark
Collateral constraint	χ	1	set as benchmark
Inflation rate	μ	1.56%	2014-19 average annual inflation
Interest rate on reserves	i_r	1.02%	Chiu et al. (2023): 2014-19 average interest rate on reserves
Reserve requirement	υ	5.60%	Chiu et al. (2023): 2014-19 average required reserve ratio
Calibrated internally			
Probability of investment	n	0.5894	2016-2018 small business loan approval rate
Bank's bargaining power (deposit)	γ	0.2547	2014-19 average deposit rate of 90-day certificates of deposit
Bank's bargaining power (loan)	θ	0.7565	2014-19 average bank prime loan rate
Production function curvature	ω	0.6068	1965-2007 money demand semi-elasticity

Table 1: Parameter Values and Targets

$\upsilon = 5.60\%$ and $i_r = 1.02\%$ to be the average of the required reserves to total balances on transaction accounts and the average interest rate on required reserves between 2014 and 2019. The inflation rate μ is chosen to match the average inflation rate 1.56% between 2014 and 2019 from FRED. Lastly, we jointly calibrate the bargaining power parameters (θ, γ) and the curvature parameter in the production function ω by targeting the average deposit rate, the average loan rate and the semi-elasticity of money demand.

The model produces the deposit rate i_d and the loan rate i_ℓ. The semi-elasticity of money demand is given by

$$\frac{\partial \ln (k_m)}{\partial i} = \frac{1}{(\omega - 1)\left[(1 + i_c) A + i - i_c - (1 - n)(1 - \gamma)(i_r - i_c)\right]}$$

since the benchmark calibration has $i_r > i_c$. The deposit rate 1.09% is the average of the interest rates (annual) on the US 90-day Certificates of Deposit between 2014 and 2019. The loan rate 4.05% is the average of the US bank prime loan rates between 2014 and 2019.[12] To find the semi-elasticity of money demand, we use the money demand data in Lucas and Nicolini (2015), which covers money demand and interest rates from 1965 to 2007. The implied semi-elasticity is -4.8688. Table 1 summarizes the calibration results.

Using the calibrated parameter values, we first examine the effects of changing the CBDC interest rate i_c in Figure 4. The Fisher equation implies $i = 0.0359$, so i_c ranges from 0 to 0.0359.

[12]The deposit rate is from FRED: IR3TCD01USA156N (3-Month or 90-day Rates and Yields: Certificates of Deposit for the United States, Percent, Annual, Not Seasonally Adjusted). The bank prime loan rate is from FRED: RIFSPBLPNA (Bank Prime Loan Rate, Percent, Annual, Not Seasonally Adjusted).

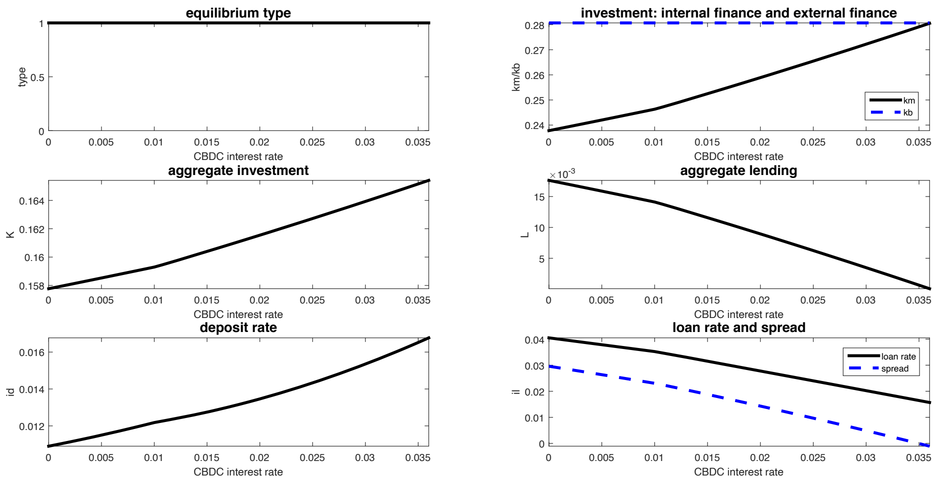

Figure 4: Effects of i_c – benchmark

As we set $\chi = 1$ and $\upsilon = 0.056$, it gives $\delta = 16.8571$ and the economy is always in the unconstrained equilibrium. Banks can hold either reserves or CBDC to satisfy the reserve requirement, depending on whether $i_c > i_r = 1.01\%$. The effects of i_c are consistent with our theoretical predictions in Proposition 1. Quantitatively, when i_c increases from 0 to 3.59%, borrowing or external finance as a fraction of investment for a banked entrepreneur decreases from 15.29% to 0.05%. Aggregate investment rises by 4.84% and aggregate lending almost reduces to 0 as entrepreneurs carry more CBDC. While we cannot show how i_d responds to i_c analytically, the calibrated example shows that i_d increases with i_c. The positive effect of i_c on i_d dominates the negative effect of ϕ on i_d. The higher i_c also reduces the spread between the loan rate and the deposit rate.

Keeping the benchmark parameter values, we also check the effects of i and δ to verify our theoretical findings in Figure 5 and Figure 6 . When i increases from 0 to 0.2, the economy is again in the unconstrained equilibrium. From (15) and other equilibrium conditions, i has the opposite effects to i_c except on the real deposit rate. This is because a change in i_c affects the nominal rates and the real rates in the same way, but a change in i could have different implications on the real rates and the nominal rates.

The theoretical results on the effects of the reserve requirement are less clear, but we illustrate these effects in Figure 6 using the calibrated example. An increase in the reserve requirement υ is equivalent to a decrease in δ. As the reserve requirement increases, the economy switches from the unconstrained equilibrium to the reserve constrained equilibrium. This is consistent with the

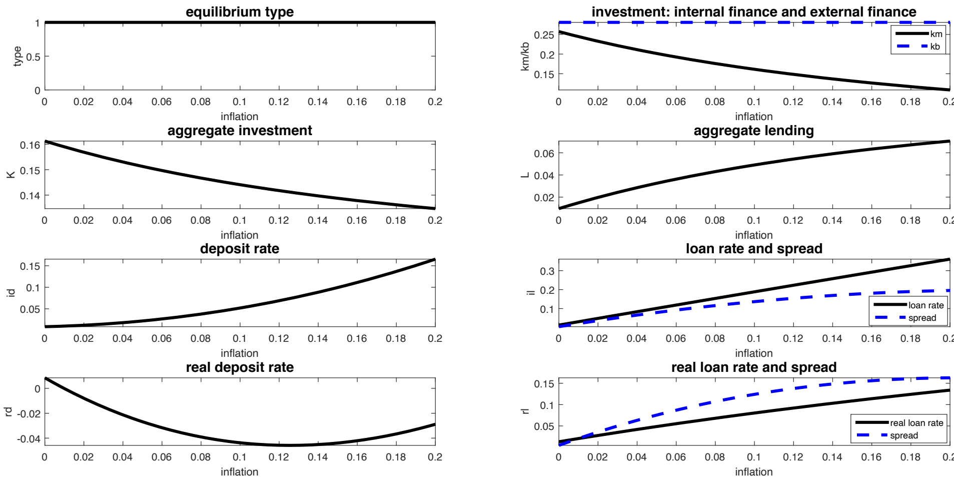

Figure 5: Effects of i − benchmark

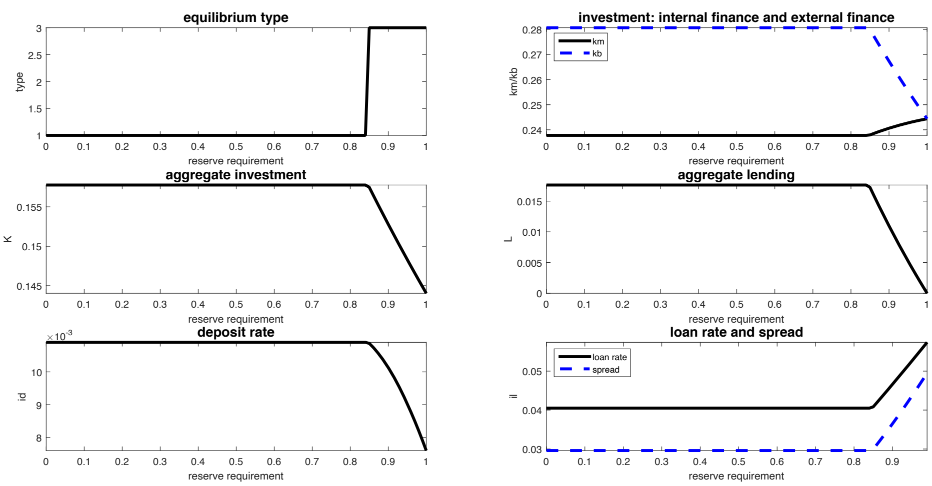

Figure 6: Effects of υ − benchmark

illustration in Figure 3 where a lower δ moves the economy from the unconstrained equilibrium (case 1) to the reserve constrained equilibrium (case 3). Our calibrated example indicates that the reserve constraint becomes binding from $\upsilon = 0.85$. In the reserved constrained equilibrium, tightening the reserve constraint encourages entrepreneurs to hold more CBDC but banked type-1 entrepreneurs have less k_b as borrowing becomes more difficult. The decline in k_b drives down both aggregate investment and aggregate lending. The lower lending leads to a lower ϕ and a lower deposit rate. However, the loan rate increases because the decrease in the size of the loan dominates the decrease in ϕ.

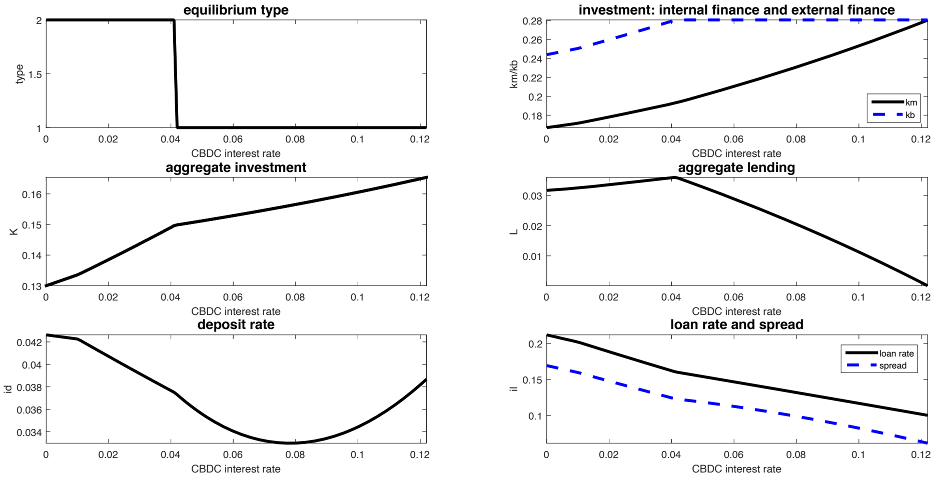

Figure 7: Effects of i_c – low χ

The benchmark calibration indicates that the economy is in the unconstrained equilibrium, and a higher i_c does not crowd out investment but hurts lending. Part of this result is consistent with the conventional wisdom that interest-bearing CBDC can lead to financial disintermediation. However, our new finding is that less lending does not imply less investment when the economy is not in the unconstrained equilibrium. In the following, we experiment with different parameter values of μ, χ and υ to investigate the effects of i_c on investment and lending in the collateral constrained equilibrium (case 2) and the reserve constrained equilibrium (case 3).

We tighten the collateral constraint by setting $\chi = 0.2$ and raising the inflation rate $\mu = 0.1$. From Figure 7, we can see that the collateral constrained equilibrium occurs when i_c is not too big, i.e., $i_c \leq 4.1\%$.[13] In this type of equilibrium, a higher i_c not only stimulates investment, but also promotes lending. As discussed earlier, the higher return of CBDC encourages entrepreneurs to hold more CBDC. With the binding collateral constraint, more CBDC implies more downpayment and more lending. Aggregate lending increases by 13.59% when i_c increases from 0 to 4.1%. The higher CBDC interest rate does not lead to financial disintermediation. As i_c increases further, entrepreneurs hold enough CBDC and the economy switches back to the unconstrained equilibrium.

When we change χ back to 1 and keep $\mu = 0.1$, but tighten the reserve constraint by setting $\delta = 0.4$, we can obtain the reserve constrained equilibrium. While we cannot show how i_c affects investment and lending analytically in the reserve constrained equilibrium, the numerical results

[13] Only changing χ to 0.2 does not alter the type of equilibrium. We need raise i which discourages entrepreneurs from holding CBDC and makes the collateral constraint binding more easily.

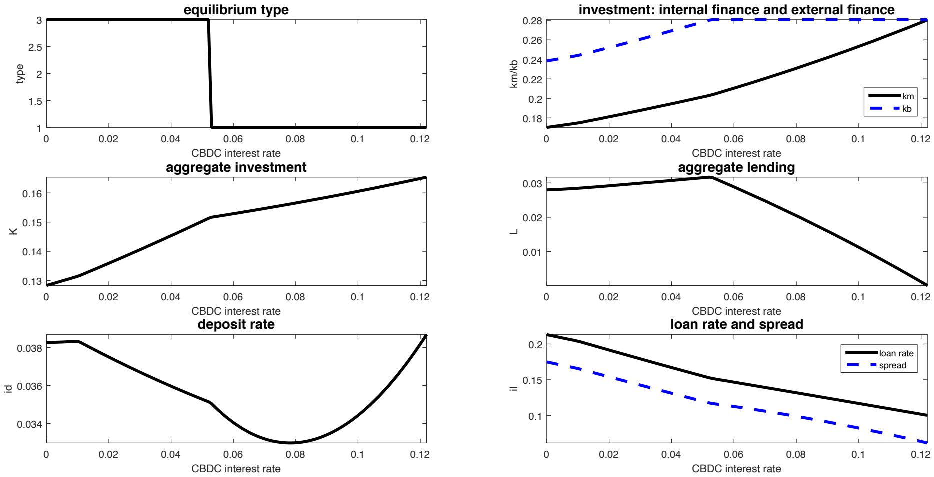

Figure 8: Effects of i_c – high υ or low δ

in Figure 8 indicate a higher i_c leads to more investment and bank lending. When the economy is in the reserve constrained equilibrium ($i_c \leq 5.2\%$), the higher return of CBDC always encourages entrepreneurs to accumulate more CBDC. In particular, investment increases by 17.9% and aggregate lending increases by 13.29%. The binding reserve constraint implies that more deposits and more reserve assets, which in turn allows banks to lend more. Thus, both lending and investment increase. Similar to the previous finding, a higher i_c eventually moves the economy to the unconstrained equilibrium.

6 Cash and CBDC

In practice, countries that consider CBDC adoption will feature the coexistence of cash and CBDC. Although few countries have issued CBDC widely, cash and CBDC may coexist for a long time, due to the consideration such as financial inclusion and disaster backup. China has confirmed that cash and e-CNY will coexist for a long time (PBOC, 2021), and furthermore added e-CNY circulation data to M0 statistics since Dec. 2022 (the ratio of e-CNY to M0 is 0.13% by the end of Dec. 2022). Even in the extreme case that cash is phased out in the long run, banknote demonetization in various countries could provide a useful reference for the transition phase. During demonetization, old series and new series of banknotes coexist in the economy for a limited period. Afterwards old banknotes can no longer circulate, but the public can still go to banks to convert them into new banknotes, within a long time.[14] Given that CBDC could bear interest, it can be challenging to have CBDC coexist with cash. Therefore, we use this section to analyze how cash and interest-bearing CBDC can coexist in a model where money and banking serve complementary roles.

The environment remains very similar to the benchmark model, except that entrepreneurs can hold a portfolio of cash and CBDC. We consider the natural case where cash and CBDC have the same value and the same growth rate μ.[15] While entrepreneurs can hold both assets, we focus on two extreme scenarios where either cash or CBDC can be accepted by banks as deposits at Stage 1. We highlight these two scenarios for two reasons. The first reason is that they facilitate the comparison between our model and existing models such as Andolfatto (2021). The second reason is that the two extreme scenarios demonstrate the importance of the relationship between CBDC and banking in determining the macroeconomic effects of CBDC.

Specifically, the first scenario is where banks can take only cash as deposits and banks can use either cash or reserves to satisfy the reserve requirement. Banks can help entrepreneurs store CBDC, but CBDC cannot be used to meet the reserve requirements. In Andolfatto (2021), individuals store CBDC at public accounts, which cannot be accessed by banks. In the second scenario, we consider the opposite banking arrangement, where banks accept only CBDC as deposits and banks use cash, CBDC or reserves to satisfy the reserve requirement. Banks do not accept cash deposits. This type of banking arrangement shares the feature of Internet banks and online banking where banks deal with electronic assets and physical cash is generally not accepted as deposits.[16]

In the following, we briefly outline how adding cash into the model and adjusting the banking arrangement change the value functions and decisions. The results from these two models help us understand the potential economic tradeoffs between cash and CBDC, and how banking arrangements or policies can affect these tradeoffs. We begin with the first scenario where only cash is accepted as deposits.

[14] For example, China started the demonetization of the 4th-series RMB banknotes in 1999, and did not complete the transition to the 5th-series RMB until 2020. Switzerland is another good example. In 2019, the Swiss National Bank confirmed it would continue to redeem without time limit the banknotes of the types issued between 1975 and 1993. This also applies to its eighth-series banknotes being recalled as of April 30, 2021.

[15] It might be interesting to consider an endogenous exchange rate between cash and CBDC, but we choose to focus on the simple and more natural scenario where they have the same value.

[16] In this scenario, we can restrict banks from holding cash to satisfy the reserve requirement if banks are strictly constrained to deal with electronic accounts. Banks would compare the CBDC interest rate and the reserve interest rate when choosing the reserve asset, which is similar to the analysis in our benchmark model. The key insight on the coexistence of cash and CBDC remains the same.

6.1 Cash Only Deposits

With cash as an additional asset, we define $\omega = z + (1 + i_c)\, z_c$ as the measure of wealth in the form of cash and CBDC for an entrepreneur at the beginning of Stage 3. In addition to the real balance of CBDC z_c, the entrepreneur has cash z in real terms. For a type-1 entrepreneur, the Stage 3 value function is

$$\begin{aligned} W_1^e(\omega, \ell + \phi, k) &= = \max_{x, \hat{z}, \hat{z}_c} \{x + \beta \mathbb{E} U^e (\hat{z}, \hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z} + (1+\mu)\hat{z}_c &= \omega - \ell - \phi + f(k) + T + \Pi. \end{aligned}$$

Converting to unconstrained optimization, we have the FOCs,

$$1 + \mu = \beta \frac{\partial \mathbb{E} U^e (\hat{z}, \hat{z}_c)}{\partial \hat{z}} = \beta \frac{\partial \mathbb{E} U^e (\hat{z}, \hat{z}_c)}{\partial \hat{z}_c}. \tag{19}$$

The expected value $\mathbb{E} U^e (\hat{z}, \hat{z}_c)$ is

$$\mathbb{E} U^e (\hat{z}, \hat{z}_c) = n U_1^e (\hat{z}, \hat{z}_c) + (1-n)\, U_0^e (\hat{z}, \hat{z}_c). \tag{20}$$

A type-0 entrepreneur has the same FOCs that determine $\hat{z}$ and $\hat{z}_c$, except that $W_0^e(\omega, d)$ has d, the amount of deposits as a state variable. Banks and suppliers have similar value functions as we derive in the benchmark model except that the asset portfolio includes both cash and CBDC.

At Stage 1 in the next period, banks and type-0 entrepreneurs are active in the deposit market, with $U_0^e(\hat{z}, \hat{z}_c) = W_0^e(\omega - d, d)$ and $U_1^e(\hat{z}, \hat{z}_c) = V_1^e(\hat{z}, \hat{z}_c)$, respectively. For banks, $U^b = (1-n)\, V^b (z_r, d) + n V^b (0, 0)$. All deposits are used as reserves $z_r = d$. If a bank does not take deposits, the bank cannot make loans. Again, $V^b(0,0) = W^b(0)$.

At Stage 2 of the loan market, a type-1 entrepreneur has

$$V_1^e(z, \hat{z}) = \frac{1-n}{n} W_1^e(\omega - p_b, \ell + \phi, k_b) + \frac{2n-1}{n} W_1^e(\omega - p_m, 0, k_m), \tag{21}$$

where p_b represents the amount of downpayment by banked entrepreneurs and p_m is the amount of payment to purchase capital by unbanked entrepreneurs. For a banked entrepreneur, p_b cannot exceed the total amount of cash and CBDC (including CBDC interest) represented by ω. The entrepreneur can use both internal finance p_b and external finance through a bank loan ℓ to purchase capital. Thus, we again have $\ell = k_b - p_b$. For an unbanked e, the amount to spend on capital p_m cannot exceed ω.

Banks can use cash or reserves to satisfy the reserve requirement. The bank's surplus from the deposit contract is $\phi + (\max\{0, i_r\} - i_d)\, d$. Otherwise, the bank's problem remains the same as before, and so does the supplier's problem.

To solve for the bargaining problems, we begin with the loan contract in the loan market taking the deposit contract (i_d, d) as given. In the loan market, a type-1 entrepreneur's surplus is $f(k_b) - k_b - \phi - [f(k_m) - p_m]$, where $p_m = \hat{z} + (1+i_c)\,\hat{z}_c = k_m$. Define $\Delta_m \equiv f(k_m) - \hat{z} - (1+i_c)\,\hat{z}_c$ as the entrepreneur's outside option where cash $\hat{z}$ and CBDC $\hat{z}_c$ are used to purchase capital. Taking $(\hat{z}, \hat{z}_c, i_d, d)$ as given, the Kalai bargaining problem remains the same as in the benchmark model except that the downpayment p_b in the constraints includes cash and CBDC. That is, $p_b \leq \hat{z} + (1+i_c)\,\hat{z}_c$.We focus on $p_b = \hat{z} + (1+i_c)\,\hat{z}_c$ because there is no benefit for the type-1 entrepreneur to keep some extra assets given that $i_c \leq i$.

We set up the Lagrangian and let (λ_1, λ_2) be the multipliers associated with the collateral constraint and the reserve constraint, respectively. The FOCs are

$$k_b : \theta\left[f'(k_b) - 1\right] = \lambda_1\left[(\theta - \chi)\, f'(k_b) + 1 - \theta\right] + \lambda_2,$$

$$\lambda_1 : k_b - \hat{z} - (1+i_c)\,\hat{z}_c + \theta\left[f(k_b) - k_b - \Delta_m\right] - \chi f(k_b) = 0,$$

$$\lambda_2 : k_b - \delta d - \hat{z} - (1+i_c)\,\hat{z}_c = 0.$$

The solution to the bargaining problem in the loan market again includes three cases, depending on the constraints. Case 1 is the unconstrained equilibrium where neither constraint binds. It gives $k_b = k^*$ and ϕ satisfies (6). Case 2 is the collateral constrained equilibrium with $\lambda_1 > 0$ and $\lambda_2 = 0$. Given $(\hat{z}, \hat{z}_c)$, the bargaining solution (k_b, ϕ, λ_1) satisfy (6), (10) and (11). Case 3 is the reserve constrained equilibrium with $\lambda_1 = 0$ and $\lambda_2 > 0$. The solution (k_b, ϕ, λ_2) is derived from (6), (12) and (13). Notice that an implicit change in these conditions is that both cash and CBDC can be used to purchase capital and $k_m = \hat{z} + (1+i_c)\,\hat{z}_c$.

In the deposit market, the Nash bargaining problem is slighted changed as banks can choose to hold cash or reserves to meet the reserve requirement, which is

$$\max_{d,i_d} \left[\phi + \left(\max\{0, i_r\} - i_d\right) d\right]^{\gamma} \left(i_d d\right)^{1-\gamma} \text{ st. } d \leq \hat{z}.$$

Again, $d = \hat{z}$ and the FOC with respect to i_d gives $i_d \hat{z} = (1-\gamma)\phi + (1-\gamma)\max\{0, i_r\}\hat{z}$.

6.1.1 General Equilibrium

To complete the description of the general equilibrium, we move back to Stage 3 to determine an entrepreneur's asset choice. The term $\mathbb{E}U^e(\hat{z}, \hat{z}_c)$ now becomes

$$\begin{aligned}\mathbb{E}U^e(\hat{z}, \hat{z}_c) &= (1-n)(1-\theta)\left[f(k_b) - k_b - f(k_m) + k_m\right] + nf(k_m) + (1-n)\left[(1+i_c)\hat{z}_c + (1+i_d)\hat{z}\right] \\ &\quad + nW_1^e(0,0,0) + (1-n)W_0^e(0,0).\end{aligned}$$

Given that $k_m = \hat{z} + (1+i_c)\hat{z}_c$, we have $\partial k_m/\partial \hat{z} = 1$ and $\partial k_m/\partial \hat{z}_c = 1 + i_c$. The expressions of $\partial(i_d\hat{z})/\partial\hat{z}$, $\partial k_b/\partial \hat{z}$ and $\partial k_b/\partial \hat{z}_c$ depend on the specific type of banking equilibrium. Only when the reserve constraint binds, a type-0 entrepreneur's deposit can affect the banking fee. Since type-0 entrepreneurs cannot deposit CBDC, their CBDC balances do not affect the terms of the deposit contract. Following similar steps as in the benchmark model, we solve for the equilibrium conditions for each type of equilibrium.

In the unconstrained equilibrium, $f'(k_b) = 1$ so that k_b does not depend on $\hat{z}$ and $\hat{z}_c$. With a slack reserve constraint, $\partial(i_d\hat{z})/\partial\hat{z} = (1-n)(1-\gamma)\max\{0, i_r\}$. It follows that (19) gives $(\hat{z}, \hat{z}_c)$ solving

$$\begin{aligned}\frac{i - i_c}{1 + i_c} &= A\left[f'(k_m) - 1\right], \\ i &= A\left[f'(k_m) - 1\right] + s_m,\end{aligned}$$

where $s_m \equiv (1-n)(1-\gamma)\max\{0, i_r\}$ represents the spread between the returns on the reserve assets and money adjusted by $(1-n)(1-\gamma)$. Both of the above conditions yield a solution for k_m.

The coexistence of cash and CBDC requires both conditions to be satisfied,

$$s_m = \frac{i_c (1+i)}{1+i_c}. \tag{22}$$

This condition implies $i_c = 0$ when $i_r \leq 0$ and banks use cash to meet the reserve requirement. Having $i_c = 0$ is the only way to make entrepreneurs hold both cash and CBDC. When $i_r > 0$, banks hold reserves that earn an interest rate i_r, which affects i_d. This additional value of cash generates a tradeoff between cash and CBDC. Condition (22) requires a specific relationship between i_c and i_r,

$$i_r = \frac{i_c (1+i)}{(1+i_c)(1-n)(1-\gamma)} \equiv i_r^*$$

such that entrepreneurs are indifferent between holding cash and CBDC.[17] The exact portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate, but $\hat{z}$ should be big enough to ensure the reserve constraint is slack.

In the collateral constrained equilibrium, k_b solves (10) and $\partial k_b / \partial \hat{z}$ remains the same as in Case 2 of the benchmark model. Given that the reserve constraint does not bind, $\partial (i_d \hat{z}) / \partial \hat{z} = (1-n)(1-\gamma) \max\{0, i_r\}$. Then (19) leads to

$$i = A\left[f'(k_m) - 1\right] + \frac{(n-A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}\left[f'(k_b) - 1\right] + s_m,$$

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + \frac{(n-A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}[f'(k_b) - 1].$$

Both conditions give the solution for k_m. Therefore, the coexistence of cash and CBDC boils down to the same condition as (22). We still have an indeterminate portfolio of $(\hat{z}, \hat{z}_c)$ and $\hat{z}$ should be big enough to ensure the non-binding reserve constraint.

In the reserve constrained equilibrium, $k_b = \delta d + k_m$. The bank's reserve d comes from another type-0 entrepreneur's deposits. Therefore, a depositor's cash balance can affect its bank's banking fee and $\partial (i_d \hat{z}) / \partial \hat{z} = \theta \delta (1-\gamma) [f'(k_b) - 1] + (1-\gamma) \max\{0, i_r\}$. Despite that $d = \hat{z}$ in equilibrium, a depositor's choice of $\hat{z}$ does not affect his/her own k_b through δd in the reserve constraint.

[17] Technically, the central bank can also adjust i to ensure the coexistence. Since i affects the return of both assets, we focus on i_c and i_r as main policy tools to understand the tradeoff between cash and CBDC.

Therefore, $\partial k_b/\partial \hat{z} = 1$ and $\partial k_b/\partial \hat{z}_c = 1 + i_c$. We can solve for $(\hat{z}, \hat{z}_c)$ from (19), which becomes

$$i = A\left[f'(k_m) - 1\right] + (n - A)\left[f'(k_b) - 1\right] + \theta\delta(1-n)(1-\gamma)\left[f'(k_b) - 1\right] + s_m, \tag{23}$$

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + (n - A)[f'(k_b) - 1]. \tag{24}$$

The coexistence of cash and CBDC implies

$$f'(k_b) - 1 = \frac{i_c(1+i)/(1+i_c) - s_m}{\theta\delta(1-n)(1-\gamma)}. \tag{25}$$

It gives the solution for k_b for $s_m < i_c(1+i)/(1+i_c)$, which implies $i_r < i_r^*$. Either (23) or (24) is used to solve for k_m. Knowing (k_m, k_b), $\hat{z}$ is found from the reserve constraint $\hat{z} = (k_b - k_m)/\delta$. In contrast to the previous two types of equilibria, the portfolio of $(\hat{z}, \hat{z}_c)$ is now determinate owing to a new tradeoff between cash and CBDC. That is, CBDC offers a return i_c, but cash now has an additional benefit by relaxing the reserve constraint. This tradeoff ensures the coexistence of cash and CBDC.

6.1.2 Policy Analysis

In the unconstrained and collateral-constrained equilibria, cash and CBDC can coexist, but the portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate.[18] Given that the central bank can adjust both i_r and i_c, it can give up one policy tool (either i_r or i_c) to allow cash and CBDC to coexist by satisfying (22). Therefore, the coexistence of cash and CBDC can be achieved through monetary policy. Proposition 4 summarizes the findings.

Proposition 4 *In an unconstrained or a collateral-constrained equilibrium, the coexistence of cash and CBDC requires (22) to hold. Either i_c or i_r cannot be an independent monetary policy tool. The portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate.*

The reserve constrained equilibrium is a more interesting case. Cash and CBDC can coexist,

[18]We focus the coexistence of cash and CBDC in our discussion. When (22) does not hold, either cash or CBDC would be driven out of existence. Suppose i_r is too low to satisfy (22). Then cash is not attractive enough for entrepreneurs to use. Entrepreneurs optimally choose to hold CBDC. However, CBDC cannot be accepted as deposits. If banks do not take deposits, they cannot issue loans. The economy will function without banks. All type-1 entrepreneurs will rely on internal finance to purchase capital. Suppose i_r is too high to satisfy (22). Then cash will dominate CBDC and becomes the only asset chosen by entrepreneurs. The economy effectively functions as the benchmark economy with $i_c = 0$.

and the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. There is no need for the central bank to sacrifice any monetary policy tool to maintain the coexistence of cash and CBDC. Compared to the previous two types of equilibria, the tradeoff between cash and CBDC depends on i_r and the extra return on cash through the banking fee. For the equilibrium to exist, i_r cannot be too big for any given i_c. The coexistence of cash and CBDC and the determinate portfolio allow us to investigate how cash and CBDC interact.

Proposition 5 *In a reserve constrained equilibrium, cash and CBDC can coexist when $i_r < i_r^*$. The portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. A higher i_c leads to a lower $\hat{z}$, a higher k_m, a lower k_b and a lower L.*

When i_c increases, the RHS of (25) increases, which implies that k_b should decrease. From (17), k_m must increase. Given that $k_b - k_m = \delta \hat{z}$ and $k_m = \hat{z} + (1 + i_c) \hat{z}_c$, $\hat{z}$ should decrease and $(1 + i_c) \hat{z}_c$ would increase. A higher i_c induces entrepreneurs to hold less cash and the fraction of CBDC in k_m increases. Type-0 entrepreneurs deposit less cash and banks issue less loans to banked type-1 entrepreneurs. In this sense, the higher return CBDC crowds out deposits, which reduces the amount of lending in the economy. This result is consistent with the common concern that CBDC might lead to financial disintermediation. The key for this result is that CBDC and banking are no longer complements because banks take cash as deposits. A higher i_c generates a redistribution effect between banked entrepreneurs and unbanked entrepreneurs. An unbanked type-1 entrepreneur purchases capital using his own cash and CBDC. The increase in $(1 + i_c) \hat{z}_c$ dominates the decrease in $\hat{z}$. The unbanked entrepreneur is able to raise k_m in response to a higher i_c. In contrast, despite that a banked entrepreneur's own portfolio allows him to purchase more capital, the reduction in bank lending leads to a lower k_b in response to a higher i_c.

6.2 CBDC Only Deposits

We now consider the second scenario where only CBDC can be accepted as deposits, but banks can use cash, CBDC or reserves to meet the reserve requirement. At Stage 3, the value functions for a type-1 entrepreneur, a type-0 entrepreneur, a bank and a supplier are the same as the ones discussed in the previous subsection. Moving to Stage 1 in the next period, we have the value function (20) for entrepreneurs where $U_1^e(\hat{z}, \hat{z}_c) = V_1^e(\hat{z}, \hat{z}_c)$ and $U_0^e(\hat{z}, \hat{z}_c) = W_0^e(\omega - d, d)$ for $d \leq \hat{z}_c$. Here d can only take the form of CBDC. For banks, $U^b = (1-n)\, V^b (z_r, d) + nV^b (0,0)\,,$ where d only comes from CBDC held by a type-0 entrepreneur. At Stage 2, a type-1 entrepreneur has the value function (21). The bank's problem and the supplier's problem remain the same as in the benchmark model.

To solve for the deposit contract and the loan contract, we begin with the bargaining problem in the loan market by taking the deposit contract as given. The bargaining problem that determines the loan contract is the same as in the previous extension. We again consider three cases, and the bargaining solutions for (p_b, k_b, ϕ) are the same as Case 1-3 in the previous extension. In the deposit market, the bargaining problem is

$$\max_{d,i_d} \left[\phi + (\max\{0, i_c, i_r\} - i_d)\, d\right]^{\gamma} \left[(i_d - i_c)\, d\right]^{1-\gamma} \text{ st. } d \le \hat{z}_c.$$

The solution is $d = \hat{z}_c$ and i_d solving

$$i_d \hat{z}_c = (1-\gamma)\phi + [\gamma i_c + (1-\gamma)\max\{0, i_c, i_r\}]\hat{z}_c. \tag{26}$$

From the loan contract, the banking fee in (26) depends on the size of deposits only when the reserve constraint binds.

6.2.1 General Equilibrium

Similarly, after solving the deposit and loan contracts, we can use the solutions to find the asset choice for $(\hat{z}, \hat{z}_c)$ at Stage 3. We have

$$\begin{aligned}\mathbb{E}U^e (\hat{z}, \hat{z}_c) = {} & (1-n)(1-\theta)\left[f(k_b) - k_b - f(k_m) + k_m\right] + nf(k_m) + (1-n)\left[\hat{z} + (1+i_d)\,\hat{z}_c\right] \\ & + nW_1^e(0,0,0) + (1-n)\, W_0^e(0,0)\,.\end{aligned}$$

In comparison with $\mathbb{E}U^e (\hat{z}, \hat{z}_c)$ in the previous extension, idle CBDC can be deposited and earns $(1+i_d)\,\hat{z}_c$. To get $\partial (i_d \hat{z}_c) / \partial \hat{z}_c$, $k_b / \partial \hat{z}$ and $\partial k_b / \partial \hat{z}_c$, again we have three cases to consider for the general equilibrium analysis.

In Case 1, the unconstrained equilibrium has $\lambda_1 = 0$**,** $\lambda_2 = 0$ and $k_b = k^*$. The solution of ϕ satisfies (6) and the term $i_d \hat{z}_c$ does not depend on the depositor's $\hat{z}_c$ since the reserve constraint does not bind. Then the FOCs for $\hat{z}$ and $\hat{z}_c$ are

$$\begin{aligned} i &= A[f'(k_m) - 1], \\ \frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + s_{rm}, \end{aligned}$$

where $s_{rm} \equiv (1-n)(1-\gamma)\max\{-i_c, 0, i_r - i_c\} / (1+i_c)$. Similar to the benchmark model, s_{rm} is the spread between the returns on the reserve assets and the CBDC interest rate, multiplied by $(1-n)(1-\gamma)$. For cash and CBDC to coexist, it requires

$$s_{rm} = -i_c (1+i) / (1+i_c). \tag{27}$$

If both $i_c \leq 0$ and $i_r \leq 0$, $s_{rm} < 0$ and (27) does not hold. Similarly, if both $i_c > 0$ and $i_r > 0$, $s_{rm} \geq 0$ and (27) does not hold. If $i_r \leq 0 \leq i_c$, $s_{rm} = 0$ and the coexistence of cash and CBDC requires $i_c = 0$ so that cash and CBDC have the same return. Lastly, if $i_c \leq 0 \leq i_r$, the coexistence of cash and CBDC requires

$$i_r = -\frac{[1 + i - (1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)} \equiv i_r^{**}.$$

This condition implies that i_r must take the opposite sign as i_c to offset the negative return on CBDC in order for cash to coexist with CBDC.

Case 2 is the collateral constrained equilibrium where $\lambda_1 > 0$ and $\lambda_2 = 0$. From the binding collateral constraint, we can derive $\partial k_b / \partial \hat{z}$ and $\partial k_b / \partial \hat{z}_c$. It follows that the FOCs for $\hat{z}$ and $\hat{z}_c$ are

$$\begin{aligned} i &= A[f'(k_m) - 1] + \frac{(n - A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}[f'(k_b) - 1], \\ \frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + \frac{(n - A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}[f'(k_b) - 1] + s_{rm}. \end{aligned}$$

The coexistence of cash and CBDC requires the same condition as (27). In both Case 1 and Case 2, the portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate and $\hat{z}_c$ should be big enough to ensure that the reserve constraint is slack.

Lastly, Case 3 is the reserve constrained equilibrium with $\lambda_1 = 0$ and $\lambda_2 > 0$. As in previous analysis with a binding reserve constraint, the depositor's CBDC balance affects the banking fee.

From (26), $\partial\left(i_d \hat{z}_c\right)/\partial \hat{z}_c = \theta\delta\left(1-\gamma\right)\left[f'\left(k_b\right)-1\right]+\left[\gamma i_c+(1-\gamma)\max\left\{0, i_c, i_r\right\}\right]$. The binding reserve constraint leads to $\partial k_b/\partial \hat{z} = 1$ and $\partial k_b/\partial \hat{z}_c = 1 + i_c$. We then derive the FOCs for $\hat{z}$ and $\hat{z}_c$ as

$$
\begin{aligned}
i &= A[f'(k_m)-1]+(n-A)[f'(k_b)-1] && (28)\\
\frac{i-i_c}{1+i_c} &= A[f'(k_m)-1]+\left[n-A+\frac{\theta\delta(1-n)(1-\gamma)}{1+i_c}\right][f'(k_b)-1]+s_{rm}. && (29)
\end{aligned}
$$

Now the coexistence of cash and CBDC implies

$$
f'(k_b)-1 = -\frac{(1+i)i_c + s_{rm}\left(1+i_c\right)}{\theta\delta(1-n)(1-\gamma)}. \quad (30)
$$

Using (30), we can solve for k_b as long as $s_{rm}\left(1+i_c\right) < -\left(1+i\right)i_c$. This condition is satisfied either both $i_c < 0$ and $i_r < i_r^{**}$. Then k_m and $\hat{z}_c$ are given by (28) and $\hat{z}_c = \left(k_b - k_m\right)/\delta$. Similar to the findings in the previous extension, the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate because CBDC has a new benefit through relaxing the reserve constraint.

6.2.2 Policy Analysis

In the unconstrained equilibrium and collateral-constrained equilibrium, (27) ensures the coexistence of cash and CBDC. The central bank must impose a negative i_c accompanied with a positive i_r, or vice versa Then entrepreneurs are indifferent in holding either of them. It also means that either i_c or i_r cannot be an independent policy tool. Proposition 6 summarizes these results.

Proposition 6 *In an unconstrained or collateral-constrained equilibrium, the coexistence of cash and CBDC requires (27) to hold. Either i_c or i_r cannot be an independent monetary policy tool. The portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate.*

As in the previous extension, the reserve constrained equilibrium has more interesting results because cash and CBDC can coexist and the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. Now CBDC has the additional value in affecting the return on deposits. The coexistence of cash and CBDC requires $i_c < 0$ and i_r to be not too big. We summarize our findings in Proposition 7.

Proposition 7 *In a reserve constrained equilibrium, cash and CBDC can coexist when $i_c < 0$ and $i_r < i_r^{**}$. The portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. A higher i_c leads to a lower $\hat{z}$, a higher $\hat{z}_c$, a lower*

k_m, a higher k_b and a higher L. A higher i_r or a higher δ leads to a higher k_m, a lower k_b and a lower L.

When i_c increases, the RHS of (30) decreases, which implies k_b should increase. It follows that k_m must decrease from (28). Then $\hat{z}_c = (k_b - k_m)/\delta$ implies that $\hat{z}_c$ increases and $k_m = \hat{z} + (1 + i_c)\hat{z}_c$ implies that $\hat{z}$ must decrease. Entrepreneurs are willing to hold more CBDC but less cash. This further leads to more lending by banks. Despite that the coexistence requires $i_c < 0$ which may seem special, the reserve constrained equilibrium still offers useful insights. A higher i_c generates a redistribution effect that allows banked entrepreneurs to purchase more k_b and unbanked entrepreneurs to purchase less k_m. In contrast to the finding in the previous extension, the higher i_c promotes banking activities and does not lead to financial disintermediation. The main reason for this result is that CBDC and banking are complements in this extension as banks accept CBDC as deposits. It again highlights that the relationship between CBDC and banking matters for understanding the effect of CBDC on banking.

A practical concern of CBDC is that the interest-bearing CBDC can potentially challenge the existence of cash. In the literature, the coexistence of cash and CBDC typically requires assumptions such as limited participation or segmented markets to prevent some agents from using certain assets. In the absence of such assumptions, we introduce economic tradeoffs between cash and CBDC so that the coexistence of cash and CBDC emerges from these economic tradeoffs. Moreover, the central bank can use proper policy tools to affect these tradeoffs.

7 Discussion and Conclusion

In this paper, we build a benchmark model with CBDC and no cash. An important feature of CBDC is that the central bank can pay interest to CBDC through digital accounts. To address the concern that the interest-bearing CBDC may cause financial disintermediation, we show that the relationship between CBDC and banking is critical. In an intermediated system of CBDC, the conversion between CBDC and deposits makes them become complements. From the benchmark model, a higher CBDC interest rate could encourage investment and bank lending through this complementality channel. The interest rate on reserves and the reserve requirement ratio can be effective policy tools.

We consider two extensions to address the coexistence of cash and CBDC. In both extensions, the coexistence of cash and CBDC does not require assumptions of limited participation or segmented markets. Since central bank's high power money includes cash, CBDC and bank reserves, the central bank has the flexibility in adjusting the interest rates of the latter two to ensure the coexistence of cash and CBDC. In the first extension where CBDC cannot be converted into deposits, and cash and banking are complements, a higher CBDC interest rate does lead to financial disintermediation whereas in the second extension where CBDC and banking are complements, a higher CBDC interest rate does not lead to financial disintermediation. These results confirm our main finding from the benchmark model that the relationship between CBDC and banking determines whether CBDC can cause financial disintermediation. Given the prevelance of an intermediated CBDC design, this complementarity between CBDC and banking should not be ignored when evaluating the macroeconomic impacts of CBDC.

CBDC is a new research topic and there are many questions left to explore, to help central banks and policy makers understand the implications of issuing CBDC. Our paper models CBDC and bank deposits as complements, which sheds light on the mainstream CBDC operational model. While other models in this literature model them as substitutes, it would be interesting to quantitatively investigate if CBDC and bank deposits tend to be complements or substitutes, when the data from CBDC pilot projects across various countries becomes available. Furthermore, it would also be interesting to investigate the implication on financial stability under this mainstream CBDC operational system. Will CBDC serve as a "flight-to-safety" vehicle, since this system potentially also facilitates fast conversion from bank deposits to CBDC? This is *not* the focus of our paper, but the solutions could be proper designs of deposit contracts (Andolfatto, 2021), or differentiating remuneration based on the amount of CBDC holdings, based on the holders being financial institutions or households (Bindseil, 2020).

Another important dimension of CBDC is about privacy: should it be anonymous or non-anonymous? At which level of anonymity should it be designed? With digital accounts of CBDC, the central bank can potentially access transaction and financial histories of all individuals. In contrast, cash transactions are anonymous. The privacy design or data sharing feature of CBDC will have important implications on consumer privacy, tax evasion and the informal economy (Wang, 2021; Xiao, 2022; Jin et al., 2023). We leave all these questions to future research.

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A Conversion Services of E-CNY and Bank Deposits

E-CNY APP

In Jan. 2022, The mobile application of China's e-CNY was launched and can be installed for payment and transfers in any smart mobilephones for users currently living in the areas where pilot projects of e-CNY were running. On August 23, 2022, there was an important updating for the app: "top up as you pay". That is, when there is insufficient balance in a digital wallet of e-CNY, this function will automatically top up from bank cards/accounts linked with the wallet.

The main purpose of this new function is to facilitate fast conversion between e-CNY and bank deposits, and consequently reinforce the complementarity between CBDC and bank deposits. For example, when a user need to pay 100 yuan but there is only 90 yuan in his e-CNY wallet, this top-up function can allow an automatic transfer of 10 yuan from the linked bank account, to complete the payment. Obviously, this new function provides more convenience for the e-CNY payment, and, more importantly, allows users to minimize the balances in the e-CNY wallet (with zero interests, for now), and instead keep the majority funds in bank accounts (with positive interests). This is also consistent with our finding that CBDC interest rate serves as a floor for the bank deposit rate in the paper.

ICBC

As one of the biggest commercial banks in China, ICBC (Industrial and Commercial Bank of China) is a pioneer to provide specific FinTech products to facilitate fast and intelligent conversion between e-CNY and bank deposits, among all commercial banks at the second tier of e-CNY. So far it has issued two products with the details as follows (source: translated from the Q&A page of ICBC e-CNY wallet).

Smart Exchange: it is a personalized service provided to individual customers, supporting automatic fund transfers from personal e-CNY wallets to ICBC accounts, to satisfy the needs of customers to earn interests from their bank deposits. Currently, there are two categories of products for this "Smart Exchange". One is "*Periodical Automatic Deposit*", which means customers can choose a fixed amount of e-CNY and automatically deposit it into the ICBC deposit account, in an agreed period. The other is "*Excess Automatic Deposit*", which means customers can keep a fixed amount of e-CNY in their personal e-CNY wallets, and automatically deposit the excess part into their ICBC deposit account every day.

Combined Payment: it means, when a customer pays with e-CNY but there are insufficient balances in his ICBC e-CNY wallet, his bank deposit can automatically be exchanged into e-CNY to complete the payment, if he already links the e-CNY wallet with his ICBC deposit account, according to the e-CNY agreement at the ICBC e-payment App.

Other Commercial Banks

Other commercial banks at the second tier of e-CNY, including Bank of China, China Construction Bank, Agricultural Bank of China, Postal Savings Bank of China, Bank of Communications, China Merchants Bank, WeBank and MyBank, all provide services for conversion between e-CNY and bank deposits (source: customer service agreements from these banks). For example, according to the e-CNY Wallet User Service Agreement, Bank of China provides the conversion services so that customers can exchange e-CNY from the digital wallet of e-CNY to bank deposits or the other way around, through the e-CNY App or the e-payment App of Bank of China.

B Proofs

Proof of Proposition 1

[1] When $i_c \geq i_r$

For the effects of changing i_c, firstly we have

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{1+i}{Af''(k_m)(1+i_c)^2} > 0 \\
\frac{\partial L}{\partial i_c} &= -(1-n)\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\frac{\partial k_m}{\partial i_c} - \hat{z}_c}{1+i_c} \lesseqgtr 0.
\end{aligned}$$

Then the effects on financial variables are,

$$
\begin{aligned}
\frac{\partial \phi}{\partial i_c} &= -\theta[f'(k_m) - 1]\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial i_d}{\partial i_c} &= 1 + (1-\gamma)\frac{d\frac{\partial \phi}{\partial i_c} - \phi\frac{\partial d}{\partial i_c}}{d^2} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{(k^* - k_m)\frac{\partial \phi}{\partial i_c} + \phi\frac{\partial k_m}{\partial i_c}}{(k^* - k_m)^2} \\
&\simeq \phi - (k^* - k_m)\,\theta\left[f'(k_m) - 1\right] \\
&= f(k^*) - f(k_m) - f'(k_m)(k^* - k_m) < 0. \\
\frac{\partial i_\ell}{\partial i_c} &= \frac{1}{1+\mu}\frac{\partial r_\ell}{\partial i_c} < 0.
\end{aligned}
$$

[2] When $i_c < i_r$

For the effects of changing i_c, firstly we have,

$$
\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{A[f'(k_m) - 1] + 1 - (1-n)(1-\gamma)}{A(1+i_c)f''(k_m)} > 0 \\
\frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k^*}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} > 0 \\
\frac{\partial L}{\partial i_c} &= (1-n)(\frac{\partial k^*}{\partial i_c} - \frac{\partial k_m}{\partial i_c}) < 0. \\
\frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\partial k_m/\partial i_c - \hat{z}_c}{1+i_c} \lesseqgtr 0.
\end{aligned}
$$

Then the effects on financial variables are,

$$
\begin{aligned}
\frac{\partial \phi}{\partial i_c} &= -\theta[f'(k_m) - 1]\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial i_d}{\partial i_c} &= \frac{(1-\gamma)}{\hat{z}_c}\frac{\partial \phi}{\partial i_c} + \frac{\gamma i_c + (1-\gamma)i_r - i_d}{\hat{z}_c}\frac{\partial \hat{z}_c}{\partial i_c} + \gamma \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{\phi - \theta[f'(k_m) - 1](k^* - k_m)}{(k^* - k_m)^2}\frac{\partial k_m}{\partial i_c} \\
&\cong \theta[f(k^*) - k^* - (f(k_m) - k_m)] - \theta(k^* - k_m)[f'(k_m) - 1] \\
&= \theta[f(k^*) - f(k_m) - f'(k_m)(k^* - k_m)] < 0 \\
\frac{\partial i_\ell}{\partial i_c} &= \frac{1}{1+\mu}\frac{\partial r_\ell}{\partial i_c} < 0.
\end{aligned}
$$

As for the effects of changing i_r, firstly we have

$$\begin{aligned}
\frac{\partial k_m}{\partial i_r} &= -\frac{(1-n)(1-\gamma)}{A(1+i_c)f''(k_m)} > 0 \\
\frac{\partial K}{\partial i_r} &= (1-n)\frac{\partial k^*}{\partial i_r} + (2n-1)\frac{\partial k_m}{\partial i_r} > 0 \\
\frac{\partial L}{\partial i_r} &= (1-n)(\frac{\partial k^*}{\partial i_r} - \frac{\partial k_m}{\partial i_r}) < 0 \\
\frac{\partial d}{\partial i_r} &= \frac{\partial \hat{z}_c}{\partial i_r} = \frac{1}{1+i_c}\frac{\partial k_m}{\partial i_r} > 0.
\end{aligned}$$

Then the effects on financial variables are,

$$\begin{aligned}
\frac{\partial i_d}{\partial i_r} &= 1-\gamma-\frac{1-\gamma}{d^2}[\theta d(f'(k_m)-1)+\frac{\phi}{1+i_r}]\frac{\partial k_m}{\partial i_r} \lesseqgtr 0 \\
\frac{\partial \phi}{\partial i_r} &= -\theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_r} < 0 \\
\frac{\partial r_\ell}{\partial i_r} &= \frac{\phi-\theta(k^*-k_m)[f'(k_m)-1]}{(k^*-k_m)^2}\frac{\partial k_m}{\partial i_r} \\
&\cong \theta[f(k^*)-k^*-(f(k_m)-k_m)]-\theta(k^*-k_m)[f'(k_m)-1] \\
&= \theta[f(k^*)-f(k_m)-f'(k_m)(k^*-k_m)] < 0 \\
\frac{\partial i_\ell}{\partial i_r} &= \frac{1}{1+\mu}\frac{\partial r_\ell}{\partial i_r} < 0.
\end{aligned}$$

For [1] $i_c \geq i_r$, or [2] $i_c < i_r$, the effects of changing i are the same as that of changing i_c, and we skip the algebra for simplicity.

Proof of Proposition 2

[1] When $i_c \geq i_r$

For the effects of changing i_c are, firstly we have

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{(1+i)[(\theta-\chi)f'(k_b)+1-\theta]}{(1+i_c)^2 D_2} > 0 \\
\frac{\partial k_b}{\partial i_c} &= -\frac{(1+i)[1+\theta(f'(k_m-1)]}{(1+i_c)^2 D_2} > 0 \\
\frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k_b}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} > 0 \\
\frac{\partial L}{\partial i_c} &= \frac{(1-n)(1+i)}{(1+i_c)^2 D_2}\{\theta[f'(k_b)-f'(k_m)]-\chi f'(k_b)\} > 0 \\
\frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\partial k_m/\partial i_c - \hat{z}_c}{1+i_c} \lesseqgtr 0,
\end{aligned}$$

where $D_2 \equiv B_2[1+\theta(f'(k_m)-1)]f''(k_b) + B_1[(\theta-\chi)f'(k_b)+1-\theta]f''(k_m) < 0$ and

$$\begin{aligned} B_1 &= \frac{\theta n[f'(k_b)-1] + A[1-\chi f'(k_b)]}{\theta[f'(k_b)-1]+1-\chi f'(k_b)} > 0 \\ B_2 &= \frac{(n-A)(1-\chi)[1+\theta(f'(k_m)-1)]}{[(\theta-\chi)f'(k_b)+1-\theta]^2} > 0. \end{aligned}$$

Then the effects on financial variables are,

$$\begin{aligned} \frac{\partial \phi}{\partial i_c} &= -\frac{\theta(1+i)}{(1+i_c)^2 D_2}\{f'(k_b) - f'(k_m) + \chi f'(k_b)[f'(k_m)-1]\} \lesseqgtr 0 \\ \frac{\partial r_\ell}{\partial i_c} &= \frac{(k_b-k_m)\partial\phi/\partial i_c - \phi(\partial k_b/\partial i_c - \partial k_m/\partial i_c)}{(k_b-k_m)^2} \lesseqgtr 0 \\ \frac{\partial i_d}{\partial i_c} &= 1 + (1-\gamma)\frac{d\frac{\partial \phi}{\partial i_c} - \phi\frac{\partial d}{\partial i_c}}{d^2} \lesseqgtr 0 \\ \frac{\partial r_d}{\partial i_c} &= \frac{1}{1+\pi}\frac{\partial i_d}{\partial i_c} \lesseqgtr 0. \end{aligned}$$

[2] **When** $i_c < i_r$

For the effects of changing i_c, firstly we have

$$\begin{aligned} \frac{\partial k_m}{\partial i_c} &= -\frac{[1+i-(1-n)(1-\gamma)(1+i_r)][(\theta-\chi)f'(k_b)+1-\theta]}{(1+i_c)^2 D_2} > 0 \\ \frac{\partial k_b}{\partial i_c} &= -\frac{[1+i-(1-n)(1-\gamma)(1+i_r)][1+\theta(f'(k_m-1)]}{(1+i_c)^2 D_2} > 0 \\ \frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k_b}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} > 0 \\ \frac{\partial L}{\partial i_c} &= \frac{(1-n)[1+i-(1-n)(1-\gamma)(1+i_r)]}{(1+i_c)^2 D_2}\{\theta[f'(k_b)-f'(k_m)] - \chi f'(k_b)\} > 0 \\ \frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\partial k_m/\partial i_c - \hat{z}_c}{1+i_c} \lesseqgtr 0. \end{aligned}$$

Then the effects on financial variables are,

$$\frac{\partial \phi}{\partial i_c} = -\frac{\theta[1+i-(1-n)(1-\gamma)(1+i_r)]}{(1+i_c)^2 D_2}\{f'(k_b) - f'(k_m) + \chi f'(k_b)[f'(k_m)-1]\} \lesseqgtr 0$$
$$\frac{\partial r_\ell}{\partial i_c} = \frac{(k_b-k_m)\partial\phi/\partial i_c - \phi(\partial k_b/\partial i_c - \partial k_m/\partial i_c)}{(k_b-k_m)^2} \lesseqgtr 0$$
$$\frac{\partial i_d}{\partial i_c} = \gamma + (1-\gamma)\frac{\hat{z}_c \frac{\partial \phi}{\partial i_c} - \phi\frac{\partial \hat{z}_c}{\partial i_c}}{\hat{z}_c^2} \lesseqgtr 0$$
$$\frac{\partial r_d}{\partial i_c} = \frac{1}{1+\pi}\frac{\partial i_d}{\partial i_c} \lesseqgtr 0.$$

As for the effects of changing i_r, firstly we have,

$$\begin{aligned}
\frac{\partial k_m}{\partial i_r} &= -\frac{(1-n)(1-\gamma)}{(1+i_c)D_2}[(\theta-\chi)f'(k_b)+1-\theta] > 0 \\
\frac{\partial k_b}{\partial i_r} &= -\frac{(1-n)(1-\gamma)}{(1+i_c)D_2}[1+\theta(f'(k_m)-1)] > 0 \\
\frac{\partial K}{\partial i_r} &= (1-n)\frac{\partial k_b}{\partial i_r} + (2n-1)\frac{\partial k_m}{\partial i_r} > 0 \\
\frac{\partial L}{\partial i_r} &= \frac{(1-n)^2(1-\gamma)}{(1+i_c)D_2}\{\theta[f'(k_b)-f'(k_m)] - \chi f'(k_b)\} > 0 \\
\frac{\partial d}{\partial i_r} &= \frac{\partial \hat{z}_c}{\partial i_r} = \frac{1}{1+i_c}\frac{\partial k_m}{\partial i_r} > 0.
\end{aligned}$$

Then the effects on financial variables are,

$$\begin{aligned}
\frac{\partial \phi}{\partial i_r} &= \frac{\theta(1-n)(1-\gamma)}{(1+i_c)D_2}\{f'(k_b) - f'(k_m) + \chi f'(k_b)[f'(k_m)-1]\} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_r} &= \frac{(k_b-k_m)\partial\phi/\partial i_r - \phi(\partial k_b/\partial i_r - \partial k_m/\partial i_r)}{(k_b-k_m)^2} \lesseqgtr 0 \\
\frac{\partial i_d}{\partial i_r} &= (1-\gamma)[1+\frac{\hat{z}_c\frac{\partial \phi}{\partial i_r} - \phi\frac{\partial \hat{z}_c}{\partial i_r}}{\hat{z}_c^2}] \lesseqgtr 0 \\
\frac{\partial r_d}{\partial i_r} &= \frac{1}{1+\mu}\frac{\partial i_d}{\partial i_r} \lesseqgtr 0.
\end{aligned}$$

Similarly, for simplicity, we skip the algebra for the effects of changing i.

Proof of Proposition 3

[1] When $i_c \geq i_r$

For the effects of changing i_c, we have,

$$\begin{aligned}\frac{\partial k_m}{\partial i_c} &= \frac{\delta k_m[n-A+\theta\delta(1-n)(1-\gamma)/(1+i_c)]f''(k_b)+\Upsilon}{(1+i_c)^2 D_4} > 0 \\ \frac{\partial k_b}{\partial i_c} &= \frac{A\delta k_m f''(k_m)-(1+\frac{\delta}{1+i_c})\Upsilon}{(1+i_c)^2 D_4} \lesseqgtr 0.\end{aligned}$$

where $D_4 \equiv Af''(k_m)+[1+\delta/(1+i_c)][n-A+\theta\delta(1-n)(1-\gamma)/(1+i_c)]f''(k_b) < 0$, and $\Upsilon \equiv \theta\delta(1-n)(1-\gamma)[f'(k_b)-1]-(1+i) < 0 = -(1+i_c)(1+A(f'(k_m)-1)+(n-A)(f'(k_b)-1)) < 0$, using (17).

The effects of changing i_c on K, L, i_d, i_ℓ are all ambiguous, and we skip the algebra.

[2] **When** $i_c < i_r$

Using (17) and (18), we have the comparative statics as follows,

$$\begin{bmatrix} A(1+i_c)f''(k_m) & Cf''(k_b) \\ 1+\frac{\delta}{1+i_c} & -1 \end{bmatrix} \cdot \begin{bmatrix} dk_m \\ dk_b \end{bmatrix} = \begin{bmatrix} Edi_c-(1-n)(1-\gamma)di_r \\ \frac{\delta k_m}{(1+i_c)^2}di_c \end{bmatrix},$$

where $C \equiv (1+i_c)(n-A)+\theta\delta(1-n)(1-\gamma) > 0$, $E \equiv -\{A[f'(k_m)-1]+(n-A)[f'(k_b)-1]+1-(1-n)(1-\gamma)\} < 0$. Hence, for the effects of changing i_c, we have

$$\begin{aligned}\frac{\partial k_m}{\partial i_c} &= -\frac{E+C\delta k_m f''(k_b)/(1+i_c)^2}{D_3} > 0 \\ \frac{\partial k_b}{\partial i_c} &= \frac{A\delta k_m f''(k_m)-E(1+i_c+\delta)}{(1+i_c)D_3} \lesseqgtr 0 \\ \frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = (\frac{\partial k_b}{\partial i_c}-\frac{\partial k_m}{\partial i_c})/\delta \lesseqgtr 0,\end{aligned}$$

where $D_3 \equiv -A(1+i_c)f''(k_m)-Cf''(k_b)[1+\delta/(1+i_c)] > 0$.

As for the effects of changing i_r, we have,

$$\begin{aligned}
\frac{\partial k_m}{\partial i_r} &= \frac{(1-n)(1-\gamma)}{D_3} > 0 \\
\frac{\partial k_b}{\partial i_r} &= \frac{(1-n)(1-\gamma)(1+\frac{\delta}{1+i_c})}{D_3} > 0 \\
\frac{\partial K}{\partial i_r} &= (1-n)\frac{\partial k_b}{\partial i_r} + (2n-1)\frac{\partial k_m}{\partial i_r} > 0 \\
\frac{\partial d}{\partial i_r} &= \frac{\partial \hat{z}_c}{\partial i_r} = \frac{(1-n)(1-\gamma)}{(1+i_c)D_3} > 0 \\
\frac{\partial L}{\partial i_r} &= (1-n)(\frac{\partial k_b}{\partial i_r} - \frac{\partial k_m}{\partial i_r}) \\
&= \frac{\delta(1-n)^2(1-\gamma)}{(1+i_c)D_3} > 0 \\
\frac{\partial \phi}{\partial i_r} &= \theta[f'(k_b)-1]\frac{\partial k_b}{\partial i_r} - \theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_r} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_r} &= \frac{(k_b-k_m)\frac{\partial \phi}{\partial i_r} - \phi(\frac{\partial k_b}{\partial i_r} - \frac{\partial k_m}{\partial i_r})}{(k_b-k_m)^2} \lesseqgtr 0 \\
&= \frac{[\theta(f'(k_b)-1)-r_\ell]\frac{\partial k_b}{\partial i_r} - [\theta(f'(k_m)-1)-r_\ell]\frac{\partial k_m}{\partial i_r}}{k_b-k_m} \\
\frac{\partial i_d}{\partial i_r} &= (1-\gamma)[1+\frac{d\frac{\partial \phi}{\partial i_r} - \phi\frac{\partial d}{\partial i_r}}{d^2}] \lesseqgtr 0 \\
&= \frac{1-\gamma}{\hat{z}_c}\{\hat{z}_c + [\theta(f'(k_b)-1)-r_\ell]\frac{\partial k_b}{\partial i_r} - [\theta(f'(k_m)-1)-r_\ell]\frac{\partial k_m}{\partial i_r}\}.
\end{aligned}$$